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(Revision of ANSI Y10.20-1975)
(Supersedes ANSI Y10.17-1961 (R 1988))

American National Standard Mathematical Signs and Symbols for Use in Physical Sciences and Technology

Sponsor
IEEE Standards Coordinating Committee 14 on Quantities, Units, and Letter Symbols

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Abstract: Signs and symbols used in writing mathematical text are defined. Special symbols peculiar to certain branches of mathematics, such as non-Euclidean Geometries, Abstract Algebras, Topology, and Mathematics of Finance, which are not ordinarily applied to the physical sciences and engineering, are omitted.
Keywords: letter symbol, mathematical notation, mathematical sign, mathematical symbol, mathematics, operation symbol, quantity symbol, unit symbol

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Introduction

(This Introduction is not part of ANSI/IEEE Std 260.3-1993, American National Standard Mathematical Signs and Symbols for Use in Physical Sciences and Technology.)

This Standard is a revision of ANSI Y10.20-1975, the original edition of *Mathematical Signs and Symbols for Use in Physical Sciences and Technology.* The purpose of this second edition is twofold: to serve as an authoritative national standard for mathematical notation and, as promised in the first edition, to include symbols of those lesser-known branches of mathematics that are increasingly being applied to the physical sciences. Added to this revision are signs and symbols used in Symbolic Logic, Set Theory, Arithmetic, Differential Geometry, Matrices, Probability and Statistics.

The Table of Signs and Symbols has been reorganized and the format modified by insertion of an additional column to exhibit, if appropriate, the application of each sign or symbol, with its meaning in that context, under the Description heading. The former subclause on Trigonometry has been renamed Circular Functions and now includes principal values of the inverse functions. To encourage the use of roman type, rather than italic type, to symbolize specific mathematical functions, the subclause on Special Functions in the original edition now occupies five subclauses; the total number of items being increased from 8 to 54.

Incorporated in this Standard is a revision of ANSI Y10.17-1961 (R 1988), *Guide for Selection of Greek Letters Used as Letter Symbols for Engineering Mathematics.* Clause 10 of this Standard supersedes ANSI Y10.17-1961 entirely.

The technical support provided by Delco Systems Operations of Delco Electronics Corporation, in the preparation of this revision, is gratefully acknowledged.

At the time this Standard was completed, the membership of the Standards Coordinating Committee 14, Quantities, Units, and Letter Symbols, consisted of:

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At the time this Standard was completed, the membership of Subcommittee 14.6, Mathematical Signs and Symbols, consisted of:

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CLAUSE PAGE

1. Scope 1

2. Letter Symbols 1

3. Alphabets and Typography 2

4. Quantity Symbols 3

5. Unit Symbols 3

6. Operation Symbols 5

7. Reference Documents 5

7.1 References 6

8. Conventions 7

9. Signs and Symbols 7

9.1 Miscellaneous Signs and Symbols 8
9.2 General Operations 9
9.3 Symbolic Logic and Set Theory 11
9.4 Arithmetic (Number Theory) 12
9.5 Elementary Functions 14
9.6 Geometry 19
9.7 Vectors 22
9.8 Matrices 23
9.9 Real Variables (Calculus) 24
9.10 Complex Variables 27
9.11 Special Functions 28
9.12 Probability and Statistics 34

10. Greek Characters 35

11. Bibliography 37

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American National Standard Mathematical Signs and Symbols for Use in Physical Sciences and Technology

1. Scope

Only signs and symbols used in writing mathematical text are contained in this Standard. Special symbols peculiar to certain branches of mathematics, such as non-Euclidean Geometries, Abstract Algebras, Topology, and Mathematics of Finance, which are not ordinarily applied to the physical sciences and engineering, have been omitted. Because there is no consensus in the literature for signs and symbols used in tensor analysis, the subject of tensors is relegated to future editions when there is general agreement among authorities in the field.

2. Letter Symbols

Letter symbols[1] include symbols for physical quantities (*quantity symbols*), symbols for units in which these quantities are measured (*unit symbols*), and symbols for operators on, and functions of, these quantities, as well as special symbols for frequently used words and phrases (*operation symbols*).

A quantity symbol is, in general, a single letter,[2] e.g., I, to represent an electric current, modified, when appropriate, by one or more subscripts or superscripts, e.g., I_{i}, to represent input current. A symbol assigned to denote a quantity in a treatise should be used consistently for that quantity throughout the work.

[1]"Letter symbol" as a technical term does not have the same meaning as either "name" or "abbreviation". An abbreviation is a letter or combination of letters (sometimes with apostrophe(s) or period) that, by convention, represents a *word* or a *name* in a particular language; hence, an abbreviation may be different in another language. A symbol represents a *quantity*, a *unit*, or an *operation*, and should be independent of language (except, by tradition, some unit symbols and their prefixes have Latin or Greek origins, and many operation symbols are Latin derivatives), e.g., the symbol for the quantity: electromotive force is "*E*", whereas the abbreviation is "emf" in English, "fem" in French, and "EMK" in German. The word for the unit of electric current "ampere" is often abbreviated "amp", but the symbol for this unit is "A". The international standard symbol for the circular function, "sine" is "sin", although. for example, the word for "sine" in Spanish is "seno".

[2]Quantity symbols comprising two letters are sometimes used for dimensionless transport parameters, e.g., Prandtl Number, "*Pr*", and Reynolds Number, "*Re*", not to be confused with the complex variables function, "Re", meaning "real part of" (qv 9.10.2, p 26).

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A unit symbol[3] is a letter or group of letters, e.g., “m” for meter(s) and “Hz” for hertz, or a special sign, such as ° for degree(s), that may be used in place of the name of the unit.

An operation symbol is a letter, a group of letters, or special sign(s) that represents a mathematical operator, a specific mathematical function or relationship, a word, or a phrase.

3. Alphabets and Typography

Letter symbols are restricted primarily to the English and Greek alphabets.[4] Script, black letter, or other special fonts to distinguish between possible conflicting uses of the same letter for different quantities should not be used.

Symbols for physical quantities, mathematical variables, indices, and general functions[5] are printed in *italic* type, e.g.,

A	area
e	eccentricity of a conic section
x, y, z	Cartesian coordinates
i, j, k	indices
$f(x)$	f function of x

Symbols used for physical units, as well as mathematical constants, specific mathematical functions, operators, and all numerals are printed in roman (upright) type, e.g.,

cm	centimeter(s)
i	imaginary unit: $\sqrt{-1}$
sin 2θ	sine of the angle: $2 \times \theta$
$a \times b + c$	a times b plus c
dx	differential of x

All punctuation,[6] including grouping symbols, such as parentheses, brackets, and braces, are also printed in roman type, e.g.,

$F(a, b; c; z)$	hypergeometric function
$[x]$	integer function of x
$[\boldsymbol{abc}]$	triple scalar product of vectors
$\{a, b\}$	LCM of a and b
$n!$	factorial n

Subscripts and superscripts are governed by the above principles. Those that are letter symbols for physical quantities, mathematical variables, or for indices are printed in italic type, whereas others are printed in roman type, e.g.,

$\sin^p x$	p^{th} power of sin x
a_{ij}, a_{45}	matrix elements
$I_{\text{i}}, I_{\text{o}}$	input, output currents
$B_x(\alpha, \beta)$	incomplete beta function

[3]It was once common to treat unit symbols in the same manner as general abbreviations, but the recommendations of the International Organization for Standardization (ISO) and many other international and national bodies concerned with standardization, emphasize the symbolic character of these designations and rigidly prescribe the manner in which they shall be treated. The concept of the unit symbol is therefore adopted in this Standard.

[4]Greek letters that are easily confused with English letters should be avoided. Clauses 8 and 10 provide a guide for selecting Greek letters to be used as symbols.

[5]The term “general functions” is used here to contrast with “specific mathematical functions”, discussed below.

[6]It should be noted that the commas, semicolons, brackets, braces, and exclamation point in these examples are mathematical operators and, consistent with the previous paragraph, should be displayed in roman type.

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πr^2 area enclosed by a circle

To indicate the vector character of a quantity, boldface type is used, italic for general vectors, roman for unit vectors and symbols for special vector functions, e.g.,

$\boldsymbol{F}_i$	force on the i^{th} element
grad $\boldsymbol{f}$	gradient of the scalar, $\boldsymbol{f}$
div$\boldsymbol{F}$	divergence of the vector, $\boldsymbol{F}$
i, j, k	orthogonal unit vectors
$\boldsymbol{k}_{\text{n}}$	normal curvature vector

The gradient symbol is boldface because its operation results in a vector, but “div” is not, as its operation results in a scalar. Ordinary italic type may be used to represent the magnitude of a general vector quantity.

4. Quantity Symbols

Quantity symbols may be used in mathematical expressions in any way consistent with good mathematical usage. The product of two scalar quantities *a* and *b* is indicated by writing *ab.* The quotient may be indicated by writing any of the following:

$\frac{a}{b}$, a/b, $a \div b$, or ab^{-1}

When more than one solidus (/) is used in an algebraic expression, grouping symbols shall be inserted to remove any ambiguity. Thus, one may write (*a*/*b*)/*c*, or *a*/[*b*/*c*], but *not a/b/c*.

Subscripts and superscripts are commonly used with quantity symbols. Several subscripts or superscripts, sometimes separated by commas, may be attached to a single letter; but, unless logical clarity dictates it, subscripts and superscriptsshould not be attached to other subscripts or superscripts.[7] A symbol that has been modified by a superscript shall be enclosed in grouping symbols before an exponent is appended.

Care should be taken not to assign the same symbol for different quantities in the same work. Use of different symbols or appending subscripts to distinguish the symbols, is recommended.

5. Unit Symbols

Unit symbols are written in lowercase letters, except the initial letter is capitalized whenever the unit is derived from a proper name.[8] The distinction between uppercase and lowercase letters shall be followed even when the symbols appear in applications where the other lettering is in uppercase style. A unit symbol is printed in roman type, the form being the same for both singular and plural. A final period (.) shall *not* be part of a unit symbol.

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[7]There are acceptable exceptions exhibited in Clause 9: Item Numbers 9.4.18, 9.4.19, 9.4.23, 9.4.24, 9.5.2.1.2, 9.8.10, 9.11.4.4.9, 9.11.4.4.10, 9.11.5.4, 9.12.10, 9.12.11.

[8]To prevent confusion with the numeral, 1, the uppercase letter, L, rather than the lowercase letter, l, is the symbol for liter. Prefixes are considered separately. Some examples of frequently used unit symbols are listed below.

picofarad	pF	decibel	dB
nanosecond	ns	volt	V
microampere	μA	kilopascal	kPa
milliliter	mL	megawatt	MW
centimeter	cm	gigahertz	GHz

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When a compound unit is formed by multiplication of two or more other units., the symbol consists of the symbols for the separate units joined by a raised dot (preferred) or by a space, e.g., N•m (or N m) for newton meter. The dot or space may be omitted in the case of familiar compounds such as watthour (symbol: Wh) if no confusion would result.[9] Hyphens shall not be used in symbols for compound units; however, exponents may be applied to unit symbols.

Should a compound unit be formed by division of one unit by another, its symbol consists of the symbols for the separate units, either separated by a solidus or conjoined using negative powers, e.g., m/s or $m \cdot s^{-1}$ for meter(s) per second. In simple cases, use of the solidus is preferred, but in no case shall more than one solidus on the same line, or a solidus followed by a product, be included in such a combination unless grouping symbols, such as parentheses, are inserted to avoid ambiguity. In complicated cases, use of negative powers is recommended.[10]

The following prefixes from the International System of Units (SI) are used to indicate decimal multiples or submultiples of units:

FACTOR	SI PREFIX	SYMBOL
10^{24}	yotta	Y
10^{21}	zetta	Z
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	P
10^{-15}	femto	f
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y

[9]It may also be omitted where adjacent symbols are separated by an exponent, as in $V \equiv kg \cdot m^2 s^{-3} A^{-1}$.
[10]The notation for products and quotients of unit symbols is intentionally made more explicit than that given in Clause 4 for quantity symbols because many unit symbols consist of more than one letter, qv Reference 3 in Subclause 7.1.

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Values that are outside the range of the above prefixes should be expressed with powers of ten applied to the base unit(s). In any case, compound prefixes shall not be used, e.g.,

tera	T	*not*	megamega	MM,
giga	G	*not*	kilomega	kM,
nano	n	*not*	millimicro	mμ,
pico	p	*not*	micromicro	μμ.

6. Operation Symbols

In this Standard “operation symbols” include signs and symbols for mathematical operators, specific mathematical functions, mathematical relations between quantities, and precisely defined symbols for words and phrases commonly used in mathematical works; e.g., the operation symbol, ∇, represents

$$\mathrm{i}\frac{\partial}{\partial x} + \mathrm{j}\frac{\partial}{\partial y} + \mathrm{k}\frac{\partial}{\partial z}$$

where the signs, ∂, +, and — (division sign) are also operation symbols, and **i, j, k**, as well as *x, y, z*, are quantity symbols. Other examples of operation symbols are “exp” for the exponential function and the symbol “∋”, which is commonly used in place of the phrase “such that”.

A compilation of operation symbols is listed in Clause 9, with their definitions and, where appropriate, typical use of the symbol and a description of its meaning in that application.

7. Reference Documents

The signs and symbols tabulated in Clause 9 are, for the most part, in accord with general usage. The original edition of this Standard and Reference 1[11] generally follow the rules stated in Clause 3 regarding the use of roman type for symbols to designate mathematical constants, operators, and specific mathematical functions. However, the other references do *not* generally conform to those precepts, often using *e* for the base of natural logarithms, *i* for the imaginary unit, $\sqrt{-1}$, *d* for the differential operator, and, among other symbols for functions, $J_n(z)$ for the Bessel function of the 1st kind of order *n.*

In addition to the references below and the handbooks listed in the bibliography, many modern textbooks were consulted to obtain a consensus for each sign and symbol; the font character being changed whenever necessary, to comply with the aforementioned rules. In some cases subjective judgment was used, but in all cases the main concern was to avoid ambiguity.

Although most of the signs and symbols depicted in Reference 1 agree in meaning with those of this Standard, there are some differences dictated by tradition in the United States. For example, the International Organization for Standardization (ISO) prefers a comma (,) to denote the decimal sign; however, in the United States and some other countries, a dot on the baseline (.) is used.

[11]Information on references can be found in Subclause 7.1.

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In general, this Standard favors the notation, definition, and meaning given in Reference 2 wherever disagreement exists with Reference 1. Since Reference 2 was published by the National Bureau of Standards[12] and is a standard reference, even for other references, it is appropriate that this Standard be consistent with Reference 2 except where type styles need to be modified to conform to the rules in Clause 3 of this Standard. Where an alternative sign or symbol displayed in Reference I agrees with this Standard, the two Standards are assumed to be in accord.[13] Those symbols in Reference 1 that belong to special fonts, violating the principles of Clause 3, are not included in this Standard.

Some minor discrepancies between ISO and symbols of this Standard (ANS) exist for circular and hyperbolic cotangents as well as their inverses (ISO 11–9; ANS 9.5.2.2, 9.5.2.3). In this Standard the cotangent functions are symbolized by “ctn” to make them more distinguishable from “cos” than “cot”. Then each of the six basic three-letter symbols has no more than one letter positionally in common with any of the other five. Also, the valid ranges in the principal values of the inverse secants and cosecants for negative arguments differ in the two Standards.

Two items in Reference 1, viz, ISO 11–11.3 and 11–13.4, use italic type, where, by the rules of Clause 3, roman type is mandatory; qv ANS 9.8.4 and 9.7.1, respectively. The combinatorial symbol, ISO item 11–6.16, uses subscripts and superscripts for arguments, but to accommodate complicated literal values for them, an ordered pair is used in this Standard, qv ANS 9.5.1.18.

The symbol, э, in Reference 1 (11–4.3) has the meaning “contains the element”, but means “such that” in ANS (9.3.7). Although zero belongs to the set, N, in Reference 1 (11–4.9), in ANS (9.3.15) the set, N, consists only of the natural numbers, i.e., the positive integers, which does not include zero. The ISO symbol for equality by definition (11–5.3) is language dependent (cf ANS 9.2.16). The order of the arguments in the symbols for elliptic integrals, as well as the sign of the characteristic, *n*, are different for the two Standards. Although the symbols used for polar, cylindrical, and spherical coordinates in Reference I are the same as those used in this Standard, the definitions of all of these symbols, except for *z*, disagree in the two documents (cf ISO 11–12, ANS 9.6.2).

The most serious variances are in the meanings attached to the associated Laguerre polynomial (ISO 11–14.13; ANS 9.11.1.5) and the exponential integral (ISO 11–14.21; ANS 9.11.4.4.3). The coefficient of y in the differential equation satisfied by the Laguerre polynomial in ISO is $n - m$, where in ANS, the (equivalent) coefficient of $f(x)$ is n. The difference is reflected in the relationship between the associated Laguerre polynomial and the derivatives of the basic Laguerre polynomial, as shown by the formula in Item Number 11–14.13 of ISO 31-11:1992:

$$L_n^m(x) = \frac{d^m}{dx^m} L_n(x) \quad (m, n \in N; m \le n)$$

This equation differs from its counterpart in Item Number 9.11.1.5 of this Standard, which agrees with Reference 2, [B4],[14] and [B9]. The exponential integral given in Reference 1, according to Reference 2, [B4], and [B5], is equivalent to $-\mathrm{Ei}(-x)$ in the notation of those references and is denoted by $E_1(x)$ in Reference 2 and [B9]. Again, this Standard subscribes to the meaning given in Reference 2.

7.1 References

[1] International Standard ISO 31-11:1992. *Mathematical signs and symbols for use in the physical sciences and technology.* 2nd ed. International Organization for Standardization.

[2] Abramowitz, Milton, and Stegun, Irene A. *Handbook of Mathematical Functions*, N. B. S. Applied Mathematics Series • 55. Washington: National Bureau of Standards, 1972.

[3] ANSI / IEEE Std 260-1978 (Reaffirmed 1991). IEEE Standard Letter Symbols for Units of Measurement.

[12]The National Bureau of Standards is now known as the National Institute of Standards and Technology.
[13]Reference 1, Introduction
[14]Brackets identify treatises listed in bibliography, Clause 11.

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8. Conventions

Many conventions for quantity symbols have been adopted by authorities in various fields of the physical sciences and technology. Most authors adhere to these conventions even though they are not mandatory. Because they facilitate comprehension by the reader who is familiar with the field, conventional symbols should be used whenever feasible. Some of the more common conventions are described here for reference.

Constants are generally denoted by the first few lowercase letters of the English alphabet: *a, b, c,* ⋯, while variables are designated by the last few letters: ⋯, *x, y, z*. Integers and indices are usually indicated by the lowercase letters: *i, j, k, l, m,* and *n*. Lowercase Greek letters often symbolize dimensionless quantities, such as ratios or angles.[15] Some physical quantities are also represented by Greek letters, e.g., λ to denote wave length, ρ to indicate the density of a substance, and ω to signify the magnitude of angular velocity.[16]

When describing the equations of motion of a physical body, such as a ship, an aircraft, a missile, etc, it is customary to use reference triads, i.e., three mutually orthogonal axes oriented in inertial space, atmosphere, earth, body, etc. A generally accepted convention for orienting the body triad assigns the +*x*-axis to the forward direction in the plane of symmetry (if practicable) parallel to the waterline (or some arbitrary plane), the +*y*-axis toward starboard, normal to the plane of symmetry, and the +*z*-axis pointed in a downward direction, perpendicular to the *x*-*y* plane. General translational motion can be described along these *x, y, z* coordinate axes.[17] Speeds are usually designated by *u, v,* and *w* along *x, y,* and *z* respectively. Full six-degree-of-freedom equations require rotations about each of the three primary body axes. The rotations about the axes *x, y,* and *z* are commonly denoted by ϕ (roll), θ (pitch), and ψ (yaw), respectively, where positive rotations are in the right-handed sense, and time rates of change of ϕ, θ, and ψ are specified by *p, q,* and *r*, respectively.

9. Signs and Symbols

The following table has been categorized into the various branches of mathematics[18] to facilitate its use, primarily, but also because some signs and symbols are common to two or more branches; e.g., ≡ generally means “is equivalent to”, as described in Item Number 9.2.13, but means “is congruent to” in Arithmetic (Item Number 9.4.3). Also, Item Numbers 9.1.4, 9.4.24, and 9.11.2.2, each describe the letter symbol, “M”, but all have different meanings. However, confusion is not very likely to result from these ambiguous definitions; the meaning should be clear by the context in which it is used, if not by the subscript, superscript, or argument(s) attached to it. Some specific quantity symbols that are associated with the various branches of mathematics are also included.[19]

[15]Angles *are* ratios when measured in radians or steradians.
[16]When necessary to distinguish the parameters, subscripts are used, e.g., ρ_0 to designate air density at sea level, and ω_e for the earth's rotational rate with respect to inertial space.
[17]Correlatives of *x, y,* and *z* are sometimes specified by the lowercase Greek letters ξ, η, and ζ, respectively.
[18]Not all branches are represented; qv Clause 1.
[19]There are certain mathematical quantities that are represented by generally accepted symbols. One purpose of this Standard is to document such symbols. Some of the items in Subclauses 9.1, 9.3, 9.4, 9.5.1, 9.6.1, 9.7, 9.8, 9.10, 9.12, as well as all of Subclauses 9.6.2, 9.6.3, consist of quantity symbols.

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9.1 Miscellaneous Signs and Symbols

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.1.1	π	$A = \pi r^2$	ratio of perimeter to diameter of a circle: 3.141 592 ⋯
9.1.2	e	$\ln e = 1$	natural logarithm base: $\lim_{n \to 8}\left(1 + \frac{1}{n}\right)^n = 2.718\ 281\ \ldots$
9.1.3	γ	$\Gamma'(1) = -\gamma$	Euler's constant: $\lim_{n \to 8}\left(\sum_{k=1}^{n} \frac{1}{k} - \ln n\right) = 0.577\ 215\ \ldots$
9.1.4	M	$\lg x \equiv M \ln x$	lg e = 0.434294⋯ (qv 9.1.2, 9.5.2.1.4, 9.5.2.1.5)
9.1.5	∞		infinity; unbounded number
9.1.6	B	B_n	Bernoulli number: $\frac{(2n)!}{(2^{2n-1}-1)\pi^{2n}} \sum_{k=1}^{\infty} (-1)^{k-1} \frac{1}{k^{2n}}$
9.1.7	E	E_n	Euler number: $\frac{2^{2n+2}(2n)!}{\pi^{2n+1}} \sum_{k=1}^{\infty} (-1)^{k-1} \frac{1}{(2k-1)^{2n+1}}$
9.1.8	s	$s(n, m)$	Stirling number of the 1st kind: $s(n+1, m) \triangleq s(n, m-1) - n s(n, m)$, where $s(1, 1) \triangleq 1$; $s(n, m) \triangleq 0$ for $m \leqq 0$, $n \leqq 0$, $n < m$
9.1.9	S	$S(n, m)$	Stirling number of the 2nd kind: $S(n+1, m) \triangleq S(n, m-1) + mS(n, m)$, where $S(1, 1) \triangleq 1$; $S(n, m) \triangleq 0$ for $m \leqq 0$, $n \leqq 0$, $n < m$
9.1.10	δ	δ_j^i	Kronecker delta: 0 for $i \neq j$, 1 for $i = j$
9.1.11	.	3 210.123 456 78	radical point (termed *decimal point* when radix is *ten*) Demarcates fractional part of the digital depiction of a real number. Spaces separate digits into groups of three.
9.1.12	(,)	(a, b)	ordered pair: a, b
9.1.13	(,,)	(x, y, z)	ordered triplet: x, y, z
9.1.14	(, ,⋯,)	$(x_1, x_2, \cdots, x_n)$	ordered n-tuplet: $x_1, x_2, \cdots, x_n$

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9.2 General Operations

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.2.1	+	$p + q$	addition; a positive value; the sum: p plus q
9.2.2	–	$p - q$	subtraction; a negative value; the difference: p minus q
9.2.3	±	$p \pm q$	plus or minus; the sum or difference of p and q
9.2.4	∓	$\mp pq$	minus or plus; the difference or sum of the product pq, e.g., $p^3 \pm q^3 \equiv (p \pm q)(p^2 \mp pq + q^2)$; ambiguous signs resolved by using either all upper signs or all lower signs throughout the expression
9.2.5		pq	multiplication; the product: p times q
9.2.6	·	$p \cdot q$	multiplication; the product: p times q (cf 9.7.4)
9.2.7	×	$p \times q$	multiplication; the product: p times q (cf 9.7.5)
9.2.8	–	$\frac{p}{q}$	division; the quotient: p divided by q
9.2.9	/	p/q	division; the quotient: p divided by q
9.2.10	÷	$p \div q$	division; the quotient: p divided by q
9.2.11	=	$p = q$	conditional equality; p is conditionally equal to q
9.2.12	≠	$p \neq q$	conditional inequality; p is not equal to q
9.2.13	≡	$p^2 \equiv pp$	unconditional equality; p^2 is equivalent to pp (cf 9.4.3)
9.2.14	≢	$\sqrt{p^2} \not\equiv p$	$\sqrt{p^2}$ is *not* equivalent to p (cf 9.4.4) (NB for real p: $\sqrt{p^2} \equiv \lvert p \rvert$, qv 9.2.31)
9.2.15	≈	$p \approx q$	approximate equality; p is approximately equal to q
9.2.16	$\stackrel{\Delta}{=}$	$p \stackrel{\Delta}{=} q$	definition; p and q are defined to be equal
9.2.17	$\triangleq$	$p \triangleq q$	correspondence; p corresponds to q

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9.2 General Operations (continued)

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.2.18	$\propto$	$p \propto q$	proportionality; *p* is proportional to *q*
9.2.19	<	$p < q$	inequality; *p* is less than *q*
9.2.20	>	$p > q$	inequality; *p* is greater than *q*
9.2.21	$\leq$	$p \leq q$	*p* is less than or equal to *q*
9.2.22	$\geq$	$p \geq q$	*p* is greater than or equal to *q*
9.2.23	$\ll$	$p \ll q$	*p* is much less than *q*
9.2.24	$\gg$	$p \gg q$	*p* is much greater than *q*
9.2.25	′	p'	prime; correlative of *p* (cf 9.9.16); also used with numerals to denote arc minute(s)
9.2.26	″	p''	double prime; correlative of *p*′ (cf 9.9.17); also used with numerals to denote arc second(s)
9.2.27	()	$(p + q)/r$	parentheses; the sum, *p* plus *q*, divided by *r*
9.2.28	[]	$[p + q]/r$	brackets; the sum, *p* plus *q*, divided by *r*
9.2.29	{}	$\{p + q\}/r$	braces; the sum, *p* plus *q*, divided by *r*
9.2.30	$\overline{\quad}$	$\overline{p + q}/r$	vinculum; the sum, *p* plus *q*, divided by *r*
9.2.31	| |	$\lvert p \rvert$	absolute value (magnitude) of *p* (cf 9.8.7, 9.10.6)
9.2.32	|| ||	$\lVert p \rVert$	norm of *p* (exact meaning depends on context)
9.2.33	$\oplus$	$p \oplus q$	in a postulational system, the sum, *p* plus *q*
9.2.34	$\ominus$	$p \ominus q$	in a postulational system, the difference, *p* minus *q*
9.2.35	$\otimes$	$p \otimes q$	in a postulational system, the product, *p* times *q*
9.2.36	$\oslash$	$p \oslash q$	in a postulational system, the quotient, *p* divided by *q*
9.2.37	⋯	*p, q, r,* ⋯; *z*	ellipsis; *et cetera* (e.g., 9.1.1, 9.1.14, 9.5.1.20, 9.8.1)

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9.3 Symbolic Logic and Set Theory

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.3.1	$\wedge$	$p \wedge q$	conjunction; proposition *p and* proposition *q*
9.3.2	$\vee$	$p \vee q$	disjunction (inclusive); proposition *p or* proposition *q* (includes the case $p \wedge q$)
9.3.3	$\veebar$	$p \veebar q$	disjunction (exclusive); proposition *p or* proposition *q* (excludes the case $p \wedge q$)
9.3.4	$\neg$	$\neg p$	negation; the negative of proposition *p*
9.3.5	$\Rightarrow$	$p \Rightarrow q$	implication; the truth of proposition p implies proposition *q* is also true
9.3.6	$\Leftrightarrow$	$p \Leftrightarrow q$	equivalence; propositions *p* and *q* are equivalent
9.3.7	$\ni$		such that
9.3.8	{ , , ⋯, }	$\{a_1, a_2, \cdots, a_n\}$	the set containing the elements a_i, $i = 1, 2, \cdots n$
9.3.9	$\in$	$a \in A$	*a* is an element (member) of set *A*
9.3.10	$\notin$	$b \notin A$	*b* is *not* an element (member) of set *A*
9.3.11	$\exists$	$\exists\, a$	there exists an element, *a*; $\exists!\, a$: there exists a *unique* element, *a*
9.3.12	$\forall$	$\forall a_i$	for all elements, a_i, $i = 1, 2, 3, \cdots, n$
9.3.13	$\varnothing$		the null (empty) set: the set that contains no elements
9.3.14	P		the set of prime numbers: {2, 3, 5, 7, 11, 13, ⋯}
9.3.15	N		the set of natural numbers: {1, 2, 3, 4, 5, 6, ⋯}
9.3.16	Z		the set of integers: $\{-n, 0, +n\}$, $n \in$ N
9.3.17	Q		the set of rational numbers: $\{m/n\}$, $m \in$ Z, $n \in$ N
9.3.18	R		the set of real numbers: $\{r\}$, $-\infty < r < +\infty$
9.3.19	C		the set of complex numbers: $\{p + iq\}$, $p \in$ R, $q \in$ R

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9.3 Symbolic Logic and Set Theory (continued)

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.3.20	=	$A = B$	coincidence; the sets A and B coincide
9.3.21	$\supset$	$A \supset B$	the set A contains B as a proper subset of A: $\exists\, a \in A \ni a \notin B$
9.3.22	$\supseteq$	$A \supseteq B$	the set A contains B as a subset (A and B may coincide)
9.3.23	$\subset$	$A \subset B$	the set B contains A as a proper subset of B: $\exists\, b \in B \ni b \notin A$
9.3.24	$\subseteq$	$A \subseteq B$	the set B contains A as a subset (A and B may coincide)
9.3.25	$\cup$	$A \cup B$	the union of set A and set B: $C \ni \forall c \in C : c \in A \vee c \in B$
9.3.26	$\bigcup$	$\bigcup_{i=1}^{n} A_i$	the union of all sets, $A_i : A_1 \cup A_2 \cup \cdots \cup A_n$
9.3.27	$\cap$	$A \cap B$	the intersection of set A and set B: $C \ni \forall c \in C : c \in A \wedge c \in B$
9.3.28	$\bigcap$	$\bigcap_{i=1}^{n} A_i$	the intersection of all sets, $A_i : A_1 \cap A_2 \cap \cdots \cap A_n$

9.4 Arithmetic (Number Theory)

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.4.1	$\mid$	$a \mid b$	a is a divisor of b, i.e., b / a is an integer
9.4.2	$\nmid$	$a \nmid b$	a is *not* a divisor of b, i.e., b / a is *not* an integer
9.4.3	$\equiv$ (mod)	$a \equiv b \pmod{m}$	a is congruent to b, *modulo* m ; i.e., $m \mid (a - b)$
9.4.4	$\not\equiv$ (mod)	$a \not\equiv b \pmod{m}$	a is *not* congruent to b, *modulo* m; i.e., $m \nmid (a - b)$
9.4.5	(,)	(a, b)	the greatest common divisor of a and b

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9.4 Arithmetic (continued)

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.4.6	{ , }	$\{a, b\}$	the least common multiple of a and b
9.4.7	U	U(n)	the least common multiple of first n positive integers
9.4.8	d	d(n)	the number of divisors of n : $\sigma_0(n)$ (qv 9.4.10)
9.4.9	σ	$\sigma(n)$	the sum of the divisors of n: $\sigma_1(n)$ (qv 9.4.10)
9.4.10	σ_m	$\sigma_m(n)$	the sum of the m^{th} powers of the divisors of n
9.4.11	p	p_n	$p \in$ P; the n^{th} prime number ($p_i < p_{i+1}$) (qv 9.3.14)
9.4.12	Ω	$\Omega(n)$	the total number of prime factors of n
9.4.13	ω	$\omega(n)$	the number of *different* prime factors of n
9.4.14	π	$\pi(n)$	pi function: the number of prime numbers $\leqq n$
9.4.15	ϑ	$\vartheta(n)$	theta function: $\Sigma \ln p$ for all prime numbers, $p \leqq n$
9.4.16	Λ	$\Lambda(n)$	lambda function: $\ln p$ for $n = p^m$, 0 for $\neq p^m$, $m \geqq 1$
9.4.17	ψ	$\psi(n)$	psi function: $\ln \mathrm{U}(n) \equiv \Sigma_{k=1}^{n} \Lambda(k)$ (qv 9.5.1.10)
9.4.18	λ	$\lambda(n)$	Liouville function: $\lambda n \triangleq + (-1)^{\Sigma k_i}$, where $n \triangleq \Pi p_i^{k_i}$
9.4.19	μ	$\mu(n)$	Möbius function: for $n \triangleq \Pi p_i^{k_i}$: $\mu(n) \triangleq (-1)^{\Sigma k_i}$ for all $k_i = 0$ or 1; if $k_i > 1$ for any $i \geqq 1$, then $\mu(n) \triangleq 0$
9.4.20	φ	$\varphi(n)$	Euler totient (phi) function: for $0 < m \leqq n$, the number of integers, $m \ni (m, n) = 1$
9.4.21	p	p(n)	the number of partitions of n
9.4.22	r	r(n)	the number of representations of $n \ni n = k^2 + m^2$
9.4.23	F	F_n	the n^{th} Fermat number: $2^{2n} + 1$
9.4.24	M	M_{p_n}	the n^{th} Mersenne number: $2^{p_n} - 1$ (qv 9.4.11)

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9.5 Elementary Functions

9.5.1 Algebraic Functions

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.5.1.1	p	x^p	x raised to the power, p: $x\ x\ x\cdots x$ (p factors for integral $p > 0$)
9.5.1.2	$-p$	x^{-p}	$1/x^p$ ($x \neq 0$ if $p \geqq 0$)
9.5.1.3	$p+q$	x^{p+q}	$x^p x^q$
9.5.1.4	pq	x^{pq}	$(x^p)^q$
9.5.1.5	$\sqrt{\ }$	$\sqrt{x}$	square root of x : $x^{\frac{1}{2}}$
9.5.1.6	$\sqrt[n]{\ }$	$\sqrt[n]{x}$	n^{th} root of x : $x^{\frac{1}{n}}$
9.5.1.7	sgn	sgn x	*signum* of x : $\frac{x}{\lvert x\rvert}$ for $x \neq 0$, 0 for $x = 0$ (qv 9.2.31)
9.5.1.8	[ ]	$[x]$	largest integer not exceeding x : $[x]$ is an integer $\ni x - 1 < [x] \leqq x$
9.5.1.9	⟨ ⟩	$\langle x \rangle$	nearest integer to x : $\left[x + \frac{1}{2}\right]$ (qv 9.5.1.8)
9.5.1.10	Σ	$\Sigma_{i=1}^{n} x_i$	sum: $x_1 + x_2 + x_3 + \cdots + x_n$ (n terms)
9.5.1.11	$\sum$	$\sum_{i=1}^{n} x_i$	sum: $x_1 + x_2 + x_3 + \cdots + x_n$ (n terms)
9.5.1.12	Π	$\Pi_{i=1}^{n} x_i$	product: $x_1\, x_2\, x_3 \cdots x_n$ (n factors)
9.5.1.13	$\prod$	$\prod_{i=1}^{n} x_i$	product: $x_1\, x_2\, x_3 \cdots x_n$ (n factors)
9.5.1.14	!	$n!$	factorial: $n! \triangleq n\,(n-1)!$, where $0! \triangleq 1$; $n! \equiv \prod_{k=1}^{n} k$ for $n \geqq 1$ (qv 9.5.1.13)
9.5.1.15	—	$0.08\overline{3},\ 0.\overline{027}$	cycle delineator for a repeating decimal number; $\frac{1}{12} \equiv 0.083\ 333\ldots,\quad \frac{1}{37} \equiv 0.027\ 027\ 027\ldots$

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9.5.1 Algebraic Functions (continued)

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.5.1.16	( )	2(3)17	common difference indicator for an arithmetic sequence; e.g., 2, 5, 8, 11, 14, 17
9.5.1.17	P	P(n, m)	permutation of n things, taken m at a time: $\mathrm{P}(n, m) \equiv \frac{n!}{(n-m)!}$ (qv 9.1.12, 9.5.1.14)
9.5.1.18	C	C(n, m)	combination of n things, taken m at a time: $\mathrm{P}(n, m) \equiv \frac{\mathrm{P}(n, m)}{m!} \equiv \frac{n!}{(n-m)!\ m!}$ (qv 9.5.1.17)
9.5.1.19	$\binom{\ }{\ }$	$\binom{n}{m}$	binomial coefficient; generating function: $(1+x)^n \equiv \sum_{m=0}^{n} \binom{n}{m} x^m;\ \binom{n}{m} \equiv \mathrm{C}(n, m)$
9.5.1.20	$\cfrac{\ }{\ +}\ \cfrac{\ }{\ +}\ \cdots$	$a_0 + \cfrac{b_0}{a_1 +}\ \cfrac{b_1}{a_2 +}\cdots$	continued fraction: $a_0 + \cfrac{b_0}{a_1 + \cfrac{b_1}{a_2 + \ddots}}$

9.5.2 Elementary Transcendental Functions

9.5.2.1 Exponential and Logarithmic Functions

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.5.2.1.1	exp	exp x	e^x (qv 9.1.2, 9.5.1.1)
9.5.2.1.2	log	$\log_b x$	logarithm (with base $b > 1$) of $x > 0$; $b^{\log_e x} \equiv x$
9.5.2.1.3	lb	lb x	binary logarithm of x: $\log_2 x$
9.5.2.1.4	lg	lg x	common (Briggsian) logarithm of x: $\log_{10} x$
9.5.2.1.5	ln	ln x	natural (Napierian) logarithm of x: $\log_e x$; $\ln_n x \triangleq \ln(\ln_{n-1} x$, where $\ln_0 x \triangleq x$

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9.5.2.2 Circular Functions

Functions in Subclause 9.5.2.2 relate Cartesian (x, y) coordinates (qv 9.6.2.1) with polar (r, θ) coordinates (qv 9.6.2.6) whose origins coincide. The argument, θ, is the angle measured around the common origin from the $+x$-axis (toward the $+y$-axis for the positive sense) to the radius vector, $\boldsymbol{r}$, whose magnitude is $r > 0$. The Cartesian coordinates of the terminus of r are the abscissa, x, and the ordinate, y.

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.5.2.2.1	sin	sin θ	sine of the angle, θ (ratio: y/r)
9.5.2.2.2	cos	cos θ	cosine of the angle, θ (ratio: x/r)
9.5.2.2.3	tan	tan θ	tangent of the angle, θ (ratio: y/x); $\theta \neq \left(k + \frac{1}{2}\right)\pi,\ k = 0, \pm 1, \pm 2, \pm 3, \ldots$
9.5.2.2.4	ctn	ctn θ	cotangent of the angle, θ (ratio: x/y); $\theta \neq k\pi,\ k = 0, \pm 1, \pm 2, \pm 3, \cdots$
9.5.2.2.5	sec	sec θ	secant of the angle, θ (ratio: r/x); $\theta \neq \left(k + \frac{1}{2}\right)\pi,\ k = 0, \pm 1, \pm 2, \pm 3, \ldots$
9.5.2.2.6	csc	csc θ	cosecant of the angle, θ (ratio: r/y); $\theta \neq k\pi,\ k = 0, \pm 1, \pm 2, \pm 3, \cdots$
9.5.2.2.7	arcsin	arcsin q	angle whose sine is q $(q^2 \leq 1)$
9.5.2.2.8	Arcsin	Arcsin q	principal value of arcsin q; restricted to $\left[-\frac{\pi}{2}, +\frac{\pi}{2}\right]$ (qv 9.9.4)
9.5.2.2.9	arccos	arccos q	angle whose cosine is q $(q^2 \leq 1)$
9.5.2.2.10	Arccos	Arccos q	principal value of arccos q; restricted to $[0, +\pi]$ (qv 9.9.4)
9.5.2.2.11	arctan	arctan q	angle whose tangent is q
9.5.2.2.12	Arctan	Arctan q	principal value of arctan q; restricted to $\left(-\frac{\pi}{2}, +\frac{\pi}{2}\right)$ (qv 9.9.1)
9.5.2.2.13	arcctn	arcctn q	angle whose cotangent is q

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9.5.2.2 Circular Functions (continued)

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.5.2.2.14	Arcctn	Arcctn q	principal value of arcctn q; restricted to $(0, +\pi)$ (qv 9.9.1)
9.5.2.2.15	arcsec	arcsec q	angle whose secant is q $(q^2 \geq 1)$
9.5.2.2.16	Arcsec	Arcsec q	principal value of arcsec q; restricted to $\left[-\pi, -\frac{\pi}{2}\right], \left[0, +\frac{\pi}{2}\right]$ (qv 9.9.3)
9.5.2.2.17	arccsc	arccsc q	angle whose cosecant is q $(q^2 \geq 1)$
9.5.2.2.18	Arccsc	Arccsc q	principal value of arccsc q; restricted to $\left[-\pi, -\frac{\pi}{2}\right], \left[0, +\frac{\pi}{2}\right]$ (qv 9.9.2)
9.5.2.2.19	vers	vers θ	versed sine of θ: $1 - \cos\theta$
9.5.2.2.20	covrs	covrs θ	coversed sine of θ or versed cosine of θ : $1 - \sin\theta$
9.5.2.2.21	exsec	exsec θ	exsecant of θ: $\sec\theta - 1$
9.5.2.2.22	hav	hav θ	haversine of θ: $\frac{1}{2}\operatorname{vers}\theta \equiv \frac{1}{2}(1-\cos\theta) \equiv \sin^2\left(\frac{\theta}{2}\right)$

9.5.2.3 Hyperbolic Functions

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.5.2.3.1	sinh	sinh θ	hyperbolic sine of θ: $\frac{1}{2}(e^{\theta} - e^{-\theta})$
9.5.2.3.2	cosh	cosh θ	hyperbolic cosine of θ: $\frac{1}{2}(e^{\theta} + e^{-\theta})$
9.5.2.3.3	tanh	tanh θ	hyperbolic tangent of θ: $\frac{\sinh\theta}{\cosh\theta}$
9.5.2.3.4	ctnh	ctnh θ	hyperbolic cotangent of θ: $\frac{\cosh\theta}{\sinh\theta}$ $(\theta \neq 0)$
9.5.2.3.5	sech	sech θ	hyperbolic secant of θ: $\frac{1}{\cosh\theta}$
9.5.2.3.6	csch	csch θ	hyperbolic cosecant of θ: $\frac{1}{\sinh\theta}$ $(\theta \neq 0)$

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9.5.2.3 Hyperbolic Functions (continued)

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.5.2.3.7	arsinh	arsinh x	reverse hyperbolic sine of x: $\ln (x + \sqrt{x^2 + 1})$
9.5.2.3.8	arcosh	arcosh x	inverse hyperbolic cosine of x: $\ln (x \pm \sqrt{x^2 - 1})$ $(x \geqq 1)$
9.5.2.3.9	Arcosh	Arcosh x	principal value of arcosh x: $\ln (x + \sqrt{x^2 - 1})$ $(x \geqq 1)$
9.5.2.3.10	artanh	artanh x	inverse hyperbolic tangent of x: $\frac{1}{2} \ln \frac{1 + x}{1 - x}$ $(x^2 < 1)$
9.5.2.3.11	arctnh	arctnh x	inverse hyperbolic cotangent of x: $\frac{1}{2} \ln \frac{x + 1}{x - 1}$ $(x^2 > 1)$
9.5.2.3.12	arsech	arsech x	inverse hyperbolic secant of x: $\ln \left(\frac{1}{x} \pm \sqrt{\frac{1}{x^2} - 1} \right)$ $(0 < x \leqq 1)$
9.5.2.3.13	Arsech	Arsech x	principal value of arsech x: $\ln \left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1} \right)$ $(0 < x \leqq 1)$
9.5.2.3.14	arcsch	arcsch x	inverse hyperbolic cosecant of x: $\ln \left(\frac{1}{x} + \sqrt{\frac{1}{x^2} + 1} \right)$ $(x \neq 0)$

9.5.2.4 Miscellaneous Elementary Transcendental Functions

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.5.2.4.1	gd	gd x	Gudermannian of x: $2 \operatorname{Arctan} e^x - \frac{\pi}{2}$
9.5.2.4.2	λ	$\lambda(\theta)$	lambda function: $\mathrm{gd}^{-1}\theta \equiv \ln (\sec \theta + \tan \theta)$

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9.6 Geometry

9.6.1 Elementary Geometry (Euclidean)

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.6.1.1	$\overline{}$	$\overline{AB}$	line segment or chord (joining points *A* and *B*)
9.6.1.2	$\longrightarrow$	$\overrightarrow{CD}$	directed line segment (from point *C* to point *D*)
9.6.1.3	$\frown$	$\overset{\frown}{EF}$	circular arc (joining points *E* and *F*)
9.6.1.4	$\parallel$	$a \parallel b$	parallel; line *a* is parallel to line *b*
9.6.1.5	$\perp$	$a \perp b$	perpendicular; line *a* is perpendicular to line *b*
9.6.1.6	$\angle$	$\angle A$	angle; angle *A*, formed by rays emanating from *A*
9.6.1.7	Δ	ΔABC	triangle; triangle with vertices at *A*, *B*, and *C*
9.6.1.8	$\square$	$\square$ABCD	square; square with vertices at *A*, *B*, *C*, and *D*
9.6.1.9	$\odot$	$\odot A(B)$	circle; circle with center at *A* passing through *B*
9.6.1.10	$\odot$	$\odot(ABC)$	circle; circle passing through *A*, *B*, and *C*
9.6.1.11	$\sim$	$\Delta ABC \sim \Delta DEF$	similarity; ΔABC is similar to ΔDEF
9.6.1.12	$\cong$	$\Delta ABC \cong \Delta DEF$	congruence; ΔABC is congruent to ΔDEF
9.6.1.13	$\between$	$\Delta ABC \between \Delta DEF$	equiangularity; ΔABC and ΔDEF are equiangular, i.e., $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$
9.6.1.14	E	$E_{\Delta ABC}$	spherical excess (of ΔABC : $\angle A + \angle B + \angle C - 180°$)
9.6.1.15	s		semi-perimeter of a triangle (plane or spherical)
9.6.1.16	s	$\perp$s	plural suffix; e.g., perpendiculars
9.6.1.17	$\therefore$		therefore; hence
9.6.1.18	$\because$		because; since
9.6.1.19	QED		*quod erat demonstrandum*: which was to he proved
9.6.1.20	QEF		*quod erat faciendum*: which was to be constructed

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9.6.2 Analytic Geometry

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.6.2.1	x, y	$P(x, y)$	rectangular (Cartesian) coordinates of a point (P) in a plane (qv 9.1.12)
9.6.2.2	h, k	$O'(h, k)$	rectangular (x, y) coordinates (respectively) of translated origin (O') in a plane (qv 9.1.12)
9.6.2.3	x, y, z	$P(x, y, z)$	rectangular (Cartesian) coordinates of a point (P) in space (qv 9.1.13)
9.6.2.4	h, k, l	$O'(h, k, l)$	rectangular (x, y, z) coordinates (respectively) of translated origin (O') in space (qv 9.1.13)
9.6.2.5	a, b, c	$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$	x, y, z intercepts; rectangular equation of a plane
9.6.2.6	α, β, γ		direction angles (from x, y, z axes, respectively)
9.6.2.7	λ, μ, ν	$\lambda^2 + \mu^2 + \nu^2 \equiv 1$	direction cosines (of angles, α, β, γ, respectively)
9.6.2.8	r, θ	$P(r, \theta)$	polar coordinates of a point (P) in a plane; transformation to rectangular coordinates: $x \triangleq r\cos\theta, y \triangleq r\sin\theta$ (qv 9.1.12)
9.6.2.9	r, θ, z	$P(r, \theta, z)$	cylindrical coordinates of a point (P) in space; transformation to rectangular coordinates: $x \triangleq r\cos\theta, y \triangleq r\sin\theta, z \triangleq z$ (qv 9.1.13)
9.6.2.10	ρ, ϕ, θ	$P(\rho, \phi, \theta)$	spherical coordinates of a point (P) in space; transformation to cylindrical coordinates: $z \triangleq \rho\cos\phi, r \triangleq \rho\sin\phi, \theta \triangleq \theta$ (qv 9.1.13)
9.6.2.11	m	$y = mx + b$	slope of a line in a plane; rectangular equation of a line
9.6.2.12	p	$y^2 = 2px$	*semi-latus rectum*; rectangular equation of a parabola p is also used to designate the length of a normal (perpendicular) to its intersection with a line (or a plane) from the origin
9.6.2.13	e	$r = \frac{p}{1 - e\cos\theta}$	eccentricity of a conic section; polar equation of a conic section (focus at the origin)

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9.6.3 Differential Geometry

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.6.3.1	s		arc length; independent intrinsic coordinate
9.6.3.2	κ	$\kappa(s)$	curvature; dependent intrinsic coordinate
9.6.3.3	τ	$\tau(s)$	torsion; dependent intrinsic coordinate
9.6.3.4	$\mathbf{t}$		unit tangent vector to a curve
9.6.3.5	$\mathbf{n}$		unit normal vector to a curve
9.6.3.6	$\mathbf{b}$		unit binormal vector to a curve: $\mathbf{b} \triangleq \mathbf{t} \times \mathbf{n}$ (qq v 9.6.3.4, 9.6.3.5, 9.7.5)
9.6.3.7	u, v, w		curvilinear coordinates
9.6.3.8	$\mathbf{N}$		unit normal vector to a surface
9.6.3.9	$E, 2F, G$		coefficients of the first fundamental form
9.6.3.10	$e, 2f, g$		coefficients of the second fundamental form
9.6.3.11	κ_1, κ_2		principal curvatures of a surface: $\kappa_1 \equiv e/E$, $\kappa_2 \equiv g/G$
9.6.3.12	M		mean curvature of a surface: $\frac{1}{2}(\kappa_1 + \kappa_2)$
9.6.3.13	K		total (Gaussian) curvature of a surface: $\kappa_1 \kappa_2$
9.6.3.14	κ_g		surface geodesic (tangential) curvature
9.6.3.15	$\boldsymbol{k}_g$		geodesic curvature vector: $\boldsymbol{k}_g \triangleq \kappa_g (\mathbf{N} \times \mathbf{t})$ (component of $\boldsymbol{k}$ in plane tangent to surface)
9.6.3.16	κ_n		surface normal curvature
9.6.3.17	$\boldsymbol{k}_n$		normal curvature vector: $\boldsymbol{k}_n \triangleq \kappa_n \mathbf{N}$ (component of $\boldsymbol{k}$ along normal to surface)
9.6.3.18	$\boldsymbol{k}$		curvature vector: $\boldsymbol{k} \triangleq \kappa\mathbf{n} \equiv \boldsymbol{k}_g + \boldsymbol{k}_n$

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9.7 Vectors

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.7.1	**i, j, k**	$a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$	**i, j, k** symbolize unit vectors parallel to, and directed along, the positive *x*, *y*, *z* Cartesian axes, respectively; $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ represents the vector directed from the origin of *x*, *y*, *z* coordinates to the diagonally opposite corner of a rectangular parallelepiped with edges *a*, *b*, *c*, respectively, along *x*, *y*, *z* axes.
9.7.2	$\boldsymbol{A}$		vector $\boldsymbol{A} \triangleq A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\,\mathbf{k}$
9.7.3	$\lvert\ \rvert$	$\lvert\boldsymbol{A}\rvert$	magnitude of $\boldsymbol{A}$: $A \triangleq \sqrt{A_x^2 + A_y^2 + A_z^2} \equiv \sqrt{A \bullet A}$
9.7.4	$\cdot$	$\boldsymbol{A} \cdot \boldsymbol{B}$	scalar (dot) product of $\boldsymbol{A}$ and $\boldsymbol{B}$: $A_xB_x + A_yB_y + A_zB_z$
9.7.5	$\times$	$\boldsymbol{A} \times \boldsymbol{B}$	vector (cross) product of $\boldsymbol{A}$ and $\boldsymbol{B}$: $(A_yB_z - A_zB_y)\,\mathbf{i} + (A_zB_x - A_xB_z)\,\mathbf{j} + (A_xB_y - A_yB_x)\,\mathbf{k}$
9.7.6	[ ]	$[\boldsymbol{abc}]$	triple scalar product: $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) \equiv (\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{c}$
9.7.7	∇		*nabla* or *del* : $\mathrm{i}\frac{\partial}{\partial x} + \mathrm{j}\frac{\partial}{\partial y} + \mathrm{k}\frac{\partial}{\partial z}$ (qv 9.9.27)
9.7.8	**grad**	**grad** *f*	gradient of *f* : $\nabla f \triangleq \frac{\partial f}{\partial x}\mathrm{i} + \frac{\partial f}{\partial y}\mathrm{j} + \frac{\partial f}{\partial z}\mathrm{k}$
9.7.9	div	div $\boldsymbol{F}$	divergence of $\boldsymbol{F}$: $\nabla \cdot \boldsymbol{F} \triangleq \nabla \cdot (F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}) \equiv \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$
9.7.10	**rot**	**rot** $\boldsymbol{F}$	curl of $\boldsymbol{F}$: $\nabla \times \boldsymbol{F} \triangleq \nabla \times (F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}) \equiv \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_z}{\partial z}\right)\mathrm{i} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\mathrm{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\mathbf{k}$
9.7.11	∇^2	$\nabla^2 f$	Laplace operator: $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial^2} \equiv \nabla \cdot \nabla$; $\nabla^2 f \equiv \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

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9.8 Matrices

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.8.1		$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{11} \\ a_{21} & a_{22} & \cdots & a_{21} \\ a_{31} & a_{32} & \cdots & a_{31} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{m1} \end{bmatrix}$	matrix, A, delineated by its elements, a_{ij}, forming a rectangular array (of numbers) having m rows and n columns
9.8.2	[ ]	$[a_{ij}]_{mn}$	matrix, A, having elements a_{ij}: $i = 1, 2, 3, \cdots, m$ (row indices), $j = 1, 2, 3, \cdots, n$ (column indices)
9.8.3	O	$A + \mathrm{O} \equiv A$	zero matrix : $\forall(i, j)$: $z_{ij} \stackrel{\Delta}{=} 0$ in $[z_{ij}]_{mn}$
9.8.4	I	$A\mathrm{I} \equiv \mathrm{I}A \equiv A$	identity matrix: $u_{ii} \stackrel{\Delta}{=} 1$, $u_{ij} \stackrel{\Delta}{=} 0$ for $i \neq j$, in $[u_{ij}]_{nn}$ (NB $u_{ij} \equiv \delta_j^{\,i}$, qv 9.1.10)
9.8.5	~	$\tilde{\mathrm{A}}$	transpose of A : $[a_{ji}]_{nm}$, where $A \stackrel{\Delta}{=} [a_{ij}]_{mn}$
9.8.6	‾	$\overline{\mathrm{A}}$	conjugate of A : $[\bar{a}_{ij}]_{mn}$, where $A \stackrel{\Delta}{=} [a_{ij}]_{mn}$ (qv 9.10.4)
9.8.7	| |	$\lvert A \rvert$	absolute value of A : $[\lvert a_{ij} \rvert]_{mn}$, where $A \stackrel{\Delta}{=} [a_{ij}]_{mn}$
9.8.8	tr	tr A	trace of A : $\Sigma_{i=1}^{n} a_{ii}$, where $\mathbf{A} \stackrel{\Delta}{=} [a_{ij}]_{nn}$
9.8.9	det	det A	determinant of A, where $A \stackrel{\Delta}{=} [a_{ij}]_{nn}$
9.8.10	cof	cof a_{ij}	cofactor of a_{ij} (in det A) : $(-1)^{i+j}$ det A_{ij}, where $A_{ij} \stackrel{\Delta}{=} [a_{k_i l_j}]_{n-1,\, n-1}$, obtained by deleting the i^{th} row and j^{th} column from $A \stackrel{\Delta}{=} [a_{kl}]_{nn}$
9.8.11	adj	adj A	adjoint of A : $\tilde{C} \stackrel{\Delta}{=} [c_{ji}]_{nn}$, where $A \stackrel{\Delta}{=} [a_{kl}]_{nn}$, and $c_{ij} \stackrel{\Delta}{=}$ cof a_{ij} in det A
9.8.12	-1	A^{-1}	inverse of A : $\dfrac{\text{adj } A}{\det A}$; $AA^{-1} \equiv A^{-1}A \equiv \mathrm{I}$ (qv 9.8.4)

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9.9 Real Variables (Calculus)

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.9.1	(,)	(a, b)	open interval, $\boldsymbol{a}$, $\boldsymbol{b}$ (of $\boldsymbol{x}$: $\boldsymbol{a} < \boldsymbol{x} < \boldsymbol{b}$)
9.9.2	(,]	$(a, b]$	half open interval, a, b (of $x : a < x \leqq b$)
9.9.3	[,)	$[\ a, b)$	half closed interval, a, b (of $x : a \leqq x < b$)
9.9.4	[,]	$[\ a, b\ ]$	closed interval, a, b (of $x : a \leqq x \leqq b$)
9.9.5	( )	$f(x)$	value of the function, f, for x
9.9.6	$\rightarrow$	$x \rightarrow a$	x approaches a without restriction
9.9.7	$\rightarrow$	$x \rightarrow a^{-}$	x approaches a from the left; i.e., $x \leqq a$
9.9.8	$\rightarrow$	$x \rightarrow a^{+}$	x approaches a from the right; i.e., $x \geqq a$
9.9.9	lim	$\lim_{x \rightarrow a} f(x)$	the limit attained by function, f, as x approaches a
9.9.10	O	$O[g(x))]$	of order no greater than $g(x)$
9.9.11	o	$o[g(x)]$	of order less than $g(x)$
9.9.12	~	$\Sigma_{i=1}^{n} i \sim n^2$	the sum of the first n positive integers is *asymptotically proportional* to n^2 as $n \rightarrow \infty$
9.9.13	$\simeq$	$\Sigma_{i=1}^{n} i \cong \frac{1}{2}n^2$	the sum of the first n positive integers is *asymptotically equal* to $\frac{1}{2}n^2$ as $n \rightarrow \infty$ [actually equal to $\frac{1}{2}n(n+1)$ for $n \geqq 1$]
9.9.14	′	$f'(a^{-})$	left hand derivative of $f(x)$ evaluated for $x = a$: $\lim_{x \rightarrow a^{-}} \{[f(x) - f(x)] / (a - x)\}$
9.9.15	′	$f'(a^{+})$	right hand derivative of $f(x)$ evaluated for $x = a$: $\lim_{x \rightarrow a^{+}} \{[f(x) - f(a)] / (x - a)\}$
9.9.16	′	$f'(a)$	derivative (slope) of $f(x)$ evaluated for $x = a$: $\lim_{x \rightarrow a} \{[f(x) - f(a)] / (x - a)\}$; $\{f'(a^{-}) = f'(a^{+})\} \Rightarrow \{f'(a) \equiv f'(a^{\pm})\}$

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9.9 Real Variables (continued)

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.9.17	$''$	$f''(a)$	2nd derivative of $f(x)$ evaluated for $x = a$: derivative of $f'(x)$ evaluated for $x = a$
9.9.18	(n)	$f^{(n)}(a)$	nth derivative of $f(x)$ evaluated for $x = a$
9.9.19	Δ	Δx	increment of x; finite difference of x
9.9.20	$\frac{\mathrm{d}}{\mathrm{d}}$	$\frac{\mathrm{d}y}{\mathrm{d}x}$	derivative of y with respect to x : $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$
9.9.21	$\frac{\mathrm{d}^n}{\mathrm{d}^n}$	$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	n^{th} derivative of y with respect to x : $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}}{\mathrm{d}x}\left(\cdots\left(\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\right)\cdots\right)\right)\right)$
9.9.22	$\cdot$	$\dot{x}$	derivative of x with respect to t (time); $\mathrm{d}x/\mathrm{d}t$; speed of x
9.9.23	$\cdot\cdot$	$\ddot{x}$	2nd derivative of x with respect to t (time); $\mathrm{d}^2x/\mathrm{d}t^2$; acceleration of x
9.9.24	(n)	$\overset{(n)}{x}$	n^{th} derivative of x with respect to t (time); $\mathrm{d}^n x/\mathrm{d}t^n$
9.9.25	$(, , \cdots)$	$f_x(a, b, \cdots)$	partial derivative of $f(x, y \cdots)$ with respect to x evaluated for $x = a, y = b, \cdots$
9.9.26	$(, , \cdots)$	$f_{xy}(a, b, \cdots)$	partial derivative of $f_x(x, y, \cdots)$ with respect to y evaluated for $x = a, y = b, \cdots$
9.9.27	$\frac{\partial}{\partial}$	$\frac{\partial u}{\partial x}$	partial derivative of $u(x, y, \cdots)$ with respect to x : $\lim_{\Delta x \to 0} \frac{u(x + \Delta x, y, \ldots) - u(x, y, \ldots)}{\Delta x}$
9.9.28	$\frac{\partial^2}{\partial\ \partial}$	$\frac{\partial^2 u}{\partial y \partial x}$	partial derivative of $\frac{\partial u}{\partial x}$ with respect to y
9.9.29	$\frac{\partial^n}{\partial^{\ n}}$	$\frac{\partial^n u}{\partial x^n}$	n^{th} partial derivative of $u(x,y,\cdots)$ with respect to x
9.9.30	d	du	total differential of $u(x,y,\cdots)$: $\frac{\partial u}{\partial x}\mathrm{d}x + \frac{\partial u}{\partial y}\mathrm{d}y + \ldots$
9.9.31	det [ ]	$\det\left[\frac{\partial u_i}{\partial x_j}\right]_{nn}$	Jacobian of $u_i(x_1, x_2, x_3, \cdots, x_n)$; $i = 1, 2, 3, \cdots, n$ (qq v 9.8.9, 9.9.27)
9.9.32	{ }	$\{x_i\}$	finite or infinite sequence of numbers, $x_1, x_2, x_3, \cdots$

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ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.9.33	inf	$\inf\{x_i\}$	*infimum* (greatest lower bound) of sequence $\{x_i\}$
9.9.34	sup	$\sup\{x_i\}$	*supremum* (least upper bound) of sequence $\{x_i\}$
9.9.35	$\int$	$\int f(x)\,dx$	indefinite integral; antiderivative of $f(x)$: $\frac{d}{dx}\left\{\int f(x)\,dx\right\} \equiv f(x)$
9.9.36	$\int$	$\int_a^b f(x)\,dx$	definite integral of $f(x)$: area defined by $a \leqq x \leqq b$ and $0 \leqq y \leqq f(x)$
9.9.37	$\int^{\infty}$	$\int_a^{\infty} f(x)\,dx$	improper integral: $\lim_{h \to \infty} \int_a^h f(x)\,dx$
9.9.38	$\fint$	$\fint_a^b f(x)\,dx$	Cauchy principal value of $\int_a^b f(x)\,dx$, where $a = -\infty$ and $b = +\infty$; or for $a < c < b$: $\lim_{x \to c^-} f(x) = \pm\infty$ and $\lim_{x \to c^+} f(x) = \mp\infty$
9.9.39	$\Big\vert$	$F(x)\Big\vert_a^b$	$F(b) - F(a)$; $F(x)$ is usually an antiderivative
9.9.40	*	$f * g$	convolution of f and g: $\int_{-\infty}^{+\infty} f(y)g(x-y)\,dy \equiv g*f$
9.9.41	$\sum^{\infty}$	$\sum_{i=1}^{\infty} u_i$	infinite sum: $\lim_{n \to \infty} \sum_{i=1}^{n} u_i$ (qv 9.5.1.11)
9.9.42	$\prod^{\infty}$	$\prod_{i=1}^{\infty} u_i$	infinite product: $\lim_{n \to \infty} \prod_{i=1}^{\infty} u_i$ (qv 9.5.1.13)

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9.10 Complex Variables

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.10.1	i	$x + iy$	imaginary unit: $\sqrt{-1}$; j is used in electrical engineering
9.10.2	Re	Rez	real part of z : $\frac{1}{2}(z + \bar{z})$; x in $z \triangleq x + iy$
9.10.3	Im	Im z	imaginary part of z: $\frac{1}{2i}(z - \bar{z})$; y in $z \triangleq x + iy$
9.10.4	—	$\bar{z}$	conjugate of z : Re z – iIm z; $x - iy$ in $z \triangleq x + iy$
9.10.5	cis	cisθ	$\cos\theta + i \sin\theta \equiv e^{i\theta}$
9.10.6	| |	$\lvert z \rvert$	absolute value of z : $\sqrt{z\bar{z}}$; $\sqrt{x^2 + y^2}$ in $z \triangleq x + iy$
9.10.7	mod	mod z	*modulus* of z : r in $z \triangleq r\ e^{i\theta}$ ($r \geqq 0$); $\lvert z \rvert$ (cf 9.10.6)
9.10.8	arg	arg z	argument of z : θ in $z \triangleq r\ e^{i\theta}$ ($r > 0$); $\arctan\frac{\mathrm{Im}z}{\mathrm{Re}z}$
9.10.9	Arg	Arg z	principal value of arg z; restricted to $(-\pi, +\pi]$
9.10.10	exp	exp z	$e^z \equiv e^x$ cis y, where $z \triangleq x + iy$ (cf 9.5.2.1.1)
9.10.11	ln	ln z	natural logarithm of z : ln $\lvert z \rvert$ + i arg z (qv 9.5.2.1.5)
9.10.12	Ln	Ln z	principal value of ln z: ln $\lvert z \rvert$ + i Arg z (cf 9.10.11)
9.10.13	sgn	sgn z	*signum* of z : $\frac{z}{\lvert z \rvert} \equiv \mathrm{cis}(\arg z)$ for $z \neq 0$, 0 for $z = 0$
9.10.14	ω	ω_n^k	k^{th}root of the n roots of unity: cis$(2k\pi/n)$, $k = 0, 1, 2, \ldots, n - 1$
9.10.15	Res	$\underset{z=a}{\mathrm{Res}}\ f(z)$	residue of $f(z)$ at $z = a$: coefficient of $(z - a)^{-1}$ in the Laurent series for $f(z)$ expanded about $z = a$
9.10.16	$\int$	$\int_C f(z)\mathrm{d}z$	line integral of $f(z)$ along curve, C, in z-plane
9.10.17	$\oint$	$\oint_C f(z)\mathrm{d}z$	line integral of $f(z)$ along closed path, C, in z-plane

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9.11 Special Functions

9.11.1 Orthogonal Polynomials

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.11.1.1	T	$T_n(x)$	Chebychev polynomial (1st kind): solution of $(1 - x^2) f''(x) - x f'(x) + n^2 f(x) = 0$
9.11.1.2	U	$U_n(x)$	Chebychev polynomial (2nd kind): solution of $(1 - x^2) f''(x) - 3x f'(x) + n (n + 2) f(x) = 0$
9.11.1.3	H	$H_n(x)$	Hermite polynomial: solution of $f''(x) - 2x f'(x) + 2n f(x) = 0$
9.11.1.4	P	$P_n^{(\alpha, \beta)}(x)$	Jacobi polynomial: solution of $(1 - x^2) f''(x) - \{\alpha - \beta + (\alpha + \beta + 2) x\} f'(x) + n (n + \alpha + \beta + 1) f(x) = 0$ $(\alpha, \beta > -1)$; $P_n^{\left(-\frac{1}{2}, -\frac{1}{2}\right)}(x) \equiv T_n(x)$, $P_n^{(0, 0)}(x) \equiv P_n(x)$, $P_n^{\left(\frac{1}{2}, \frac{1}{2}\right)}(x) \equiv U_g(x)$
9.11.1.5	L	$L_n^{(\alpha)}(x)$	Laguerre polynomial (associated): solution of $x f''(x) + (\alpha + 1 - x) f'(x) + n f(x) = 0$ $(\alpha > -1)$; $L_n^{(m)}(x) \equiv (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$, $m = 0, 1, 2, \cdots, n$; $L_n(x) \triangleq L_n^{(0)}(x)$
9.11.1.6	P	$P_n(x)$	Legendre polynomial: solution of $(1 - x^2) f''(x) - 2x f'(x) + n (n + 1) f(x) = 0$

9.11.2 Hypergeometric Functions

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.11.2.1	F	$F(a, b; c; z)$	hypergeometric function: solution of $z (1 - z) f''(z) + \{c - (a + b + 1) z\} f'(z) - ab f(z) = 0$
9.11.2.2	M	$M(a, b, z)$	confluent hypergeometric function: solution of $z f''(z) + (b - z) f'(z) - a f(z) = 0$

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9.11.3 Bessel Functions

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.11.3.1	J	$J_n(z)$	Bessel function of the 1st kind of order n : solution of $z^2 f''(z) + z f'(z) + (z^2 - n^2) f(z) = 0$
9.11.3.2	Y	$Y_n(z)$	Bessel function of the 2nd kind of order n: $\lim_{\varepsilon \to 0} \frac{J_{n+\varepsilon}(z) \cos(n+\varepsilon)\pi - J_{-(n+\varepsilon)}(z)}{\sin(n+\varepsilon)\pi}$
9.11.3.3	I	$I_n(z)$	modified Bessel function of the 1st kind of order n: solution of $z^2 f''(z) + z f'(z) - (z^2 + n^2 f(z) = 0$
9.11.3.4	K	$K_n(z)$	modified Bessel function of the 2nd kind of order n: $\frac{\pi}{2} \lim_{\varepsilon \to 0} \frac{I_{-(n+\varepsilon)}(z) - I_{n+\varepsilon}(z)}{\sin(n+\varepsilon)\pi}$
9.11.3.5	H	$H_n^{(v)}(z)$	Bessel function of the 3rd kind of order n [often called Hankel function]: $J_n(z) + i^{2v-1} Y_n(z)$; defined for $v = 1, 2$
9.11.3.6	ber	$\mathrm{ber}_n x$	Kelvin function of the 1st kind of order n (real): $\mathrm{Re}\left\{ J_n\left(i^{\frac{3}{2}} x \right) \right\}$ for real n, $x > 0$ (qv 9.10.2)
9.11.3.7	bei	$\mathrm{bei}_n x$	Kelvin function of the 1st kind of order n (imaginary): $\mathrm{Im}\left\{ J_n\left(i^{\frac{3}{2}} x \right) \right\}$ for real n, $x > 0$ (qv 9.10.3)
9.11.3.8	ker	$\mathrm{ker}_n x$	Kelvin function of the 2nd kind of order n (real): $\mathrm{Re}\left\{ i^{-n} K_n\left(i^{\frac{1}{2}} x \right) \right\}$ for real n, $x > 0$ (qv 9.10.2)
9.11.3.9	kei	$\mathrm{kei}_n x$	Kelvin function of the 2nd kind of order n (imaginary): $\mathrm{Im}\left\{ i^{-n} K_n\left(i^{\frac{1}{2}} x \right) \right\}$ for real n, $x > 0$ (qv 9.10.3)
9.11.3.10	her	$\mathrm{her}_n x$	Kelvin function of the 3rd kind of order n (real): $\mathrm{Re}\left\{ H_n^{(1)}\left(i^{\frac{3}{2}} x \right) \right\}$ for real n, $x > 0$ (qv 9.10.2)
9.11.3.11	hei	$\mathrm{hei}_n x$	Kelvin function of the 3rd kind of order n (imaginary): $\mathrm{Im}\left\{ H_n^{(1)}\left(i^{\frac{3}{2}} x \right) \right\}$ for real n, $x > 0$ (qv 9.10.3)

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9.11.4 Integral Functions

9.11.4.1 Elliptic Integrals

($\phi \triangleq$ amplitude, $\alpha \triangleq$ modular angle, $k \triangleq$ modulus, $m \triangleq$ parameter, $n \triangleq$ characteristic)

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.11.4.1.1	F	$F(\phi\backslash\alpha)$, $F(\phi, k)$, $F(\phi\|m)$	incomplete elliptic integral of the 1st kind: $\int_0^{\phi} \frac{d\theta}{\sqrt{1-\sin^2\alpha \sin^2\theta}} \equiv \int_0^{\phi} \frac{d\theta}{\sqrt{1-k^2 \sin^2\theta}}$ $\equiv \int_0^{\varphi} \frac{d\theta}{\sqrt{1-m^2 \sin^2\theta}} \equiv \int_0^{\sin\varphi} \frac{dx}{\sqrt{(1-x^2)(1-mx^2)}}$
9.11.4.1.2	K	$K(k)$	complete elliptic integral of the 1st kind: $F\left(\frac{\pi}{2}, k\right)$
9.11.4.1.3	E	$E(\phi\backslash\alpha)$, $E(\phi, k)$, $E(\phi\|m)$	incomplete elliptic integral of the 2nd kind: $\int_0^{\phi} \sqrt{1-\sin^2\alpha \sin^2\theta}\, d\theta \equiv \int_0^{\phi} \sqrt{1-k^2 \sin^2\theta}\, d\theta$ $\equiv \int_0^{\phi} \sqrt{1-m^2 \sin^2\theta}\, d\theta \equiv \int_0^{\sin\phi} \sqrt{\frac{1-mx^2}{1-x^2}}\, dx$
9.11.4.1.4	E	$E(k)$	complete elliptic integral of the 2nd kind: $E\left(\frac{\pi}{2}, k\right)$
9.11.4.1.5	Z	$Z(\phi, k)$	Jacobi zeta function: $E(\phi, k) - \frac{E(k)}{K(k)} F(\phi, k)$
9.11.4.1.6	Π	$\Pi(n; \phi\backslash\alpha)$, $\Pi(n; \phi, k)$, $\Pi(n; \phi\|m)$	incomplete elliptic integral of the 3rd kind: $\int_0^{\phi} \frac{d\theta}{(1-n \sin^2\theta)\sqrt{1-\sin^2\alpha \sin^2\theta}}$ $\equiv \int_0^{\phi} \frac{d\theta}{(1-n \sin^2\theta)\sqrt{1-k^2 \sin^2\theta}}$ $\equiv \int_0^{\varphi} \frac{d\theta}{(1-n \sin^2\theta)\sqrt{1-m \sin^2\theta}}$ $\equiv \int_0^{\sin\varphi} \frac{dx}{(1-nx^2)\sqrt{(1-x^2)(1-mx^2)}}$
9.11.4.1.7	Π	$\Pi(n, k)$	complete elliptic integral of the 3rd kind: $\Pi\left(n; \frac{\pi}{2}, k\right)$

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9.11.4.2 Jacobi Elliptic Functions (qv 9.11.4.1.1)

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.11.4.2.1	am	am u	amplitude of u: ϕ in $u \triangleq F(\phi, k)$
9.11.4.2.2	sn	sn u	$\sin \phi$ in $u \triangleq F(\phi, k)$
9.11.4.2.3	cn	cn u	$\cos \phi$ in $u \triangleq F(\phi, k)$
9.11.4.2.4	dn	dn u	$\sqrt{1 - k^2 \sin^2 \phi}$ in $u \triangleq F(\phi, k)$
9.11.4.2.5	pq	pq u	$\frac{\mathrm{pn}\, u}{\mathrm{qn}\, u}$ p, q $\triangleq$ s, c, d ; p $\neq$ q
9.11.4.2.6	np	np u	$\frac{1}{\mathrm{pn}\, u}$ p $\triangleq$ s, c, d

9.11.4.3 Gamma Functions

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.11.4.3.1	Γ	$\Gamma(x)$	gamma function (complete): $\int_0^\infty t^{x-1} e^{-t} dt \quad (x > 0)$
9.11.4.3.2	γ	$\gamma(x, u)$	incomplete gamma function: $\int_0^u t^{x-1} e^{-t} dt \quad (x > 0)$
9.11.4.3.3	Γ	$\Gamma(x, u)$	incomplete gamma function: $\Gamma(x) - \gamma(x, u)$ $\equiv \int_u^\infty t^{x-1} e^{-t} dt \quad (x > 0)$
9.11.4.3.4	ψ	$\psi(x)$	psi (digamma) function: $\frac{\Gamma'(x)}{\Gamma(x)} \equiv \frac{d[\ln \Gamma(x)]}{dx}$ $\equiv \int_0^\infty \left\{ \frac{e^{-t}}{t} - \frac{e^{-xt}}{1 - e^{-t}} \right\} dt \quad (x > 0)$
9.11.4.3.5	ψ	$\psi^{(n)}(x)$	polygamma function: $\frac{d^n \psi(x)}{dx^n} \equiv \frac{d^{n+1}[\ln \Gamma(x)]}{dx^{n+1}}$ $\equiv (-1)^{n+1} \int_0^\infty \frac{t^n e^{-xt}}{1 - t} dt \quad (x > 0)$ (qv 9.9.38)

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9.11.4.4 Miscellaneous Integral Functions

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.11.4.4.1	B	$B(\alpha, \beta)$	beta function (complete): $\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,dt \equiv \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ (qv 9.11.4.3.1)
9.11.4.4.2	B	$B_x(\alpha, \beta)$	incomplete beta function: $\int_0^x t^{\alpha-1}(1-t)^{\beta-1}\,dt \quad (0 \leqq x \leqq 1)$
9.11.4.4.3	Ei	Ei(x)	exponential integral: $\int_{-\infty}^{x} \frac{e^t}{t}\,dt \equiv -\int_{-x}^{\infty} \frac{e^{-t}}{t}\,dt \quad (x > 0)$ (qv 9.9.38)
9.11.4.4.4	li	li(x)	logarithmic integral: $\int_0^x \frac{dt}{\ln t} \equiv \mathrm{Ei}(\ln x) \quad (x > 1)$ (qv 9.11.4.4.3)
9.11.4.4.5	Si	Si(x)	sine integral: $\int_0^x \frac{\sin t}{t}dt \equiv \frac{\pi}{2} - \int_x^\infty \frac{\sin t}{t}dt$
9.11.4.4.6	Ci	Ci(x)	cosine integral: $-\int_x^\infty \frac{\cos t}{t}dt$ $\equiv \gamma + \ln x - \int_0^x \frac{1-\cos t}{t}dt \qquad (x > 0)$ (qv 9.1.3)
9.11.4.4.7	S	S(x)	Fresnel sine integral: $\int_0^x \sin\left(\frac{\pi t^2}{2}\right)dt$
9.11.4.4.8	C	C(x)	Fresnel cosine integral: $\int_0^x \cos\left(\frac{\pi t^2}{2}\right)dt$
9.11.4.4.9	erf	erf(x)	error function: $\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \equiv \frac{1}{\sqrt{\pi}}\gamma\left(\frac{1}{2}, x^2\right)$
9.11.4.4.10	erfc	erfc(x)	complementary error function: $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt$ $\mathrm{erfc}(x) \equiv 1 - \mathrm{erf}\,(x)$ (qv 9.11.4.4.9)

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9.11.5 Miscellaneous Special Functions

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.11.5.1	δ	$\delta(x)$	Dirac delta function: $\int_{-\infty}^{+\infty} f(x)\ \delta(x-a)\ \mathrm{d}x \triangleq f(a)$
9.11.5.2	B	$B_n(x)$	Bernoulli polynomial of degree n; generating function: $\frac{t\,e^{xt}}{e^t-1} \equiv \sum_{n=0}^{\infty} B_n(x)\ \frac{t^n}{n!}$ $(\lvert t\rvert < 2\pi)$; n^{th} Bernoulli number ($n = 1, 2, 3, \cdots$): $B_n \equiv (-1)^{n-1}\ B_{2n}(0)$ (cf 9.1.6)
9.11.5.3	E	$E_n(x)$	Euler polynomial of degree n; generating function: $\frac{2\,e^{xt}}{e^t+1} \equiv \sum_{n=0}^{\infty} E_n(x)\ \frac{t^n}{n!}$ $(\lvert t\rvert < \pi)$; n^{th} Euler number ($n = 0, 1, 2, \cdots$): $E_n \equiv (-1)^n 2^{2n} E_{2n}\left(\frac{1}{2}\right)$ (cf 9.1.7)
9.11.5.4	ϑ	$\vartheta_n(z, q)$	Jacobi theta functions: $n = 1, 2, 3, 4$; $q \triangleq$ nome: $\vartheta_1(z, q) \triangleq 2\Sigma_{k=0}^{\infty}(-1)^k q^{\left(k+\frac{1}{2}\right)^2} \sin(2k+1)z$, $\vartheta_2(z, q) \triangleq 2\Sigma_{k=0}^{\infty} q^{\left(k+\frac{1}{2}\right)^2} \cos(2k+1)z$, $\vartheta_3(z, q) \triangleq 1 + 2\Sigma_{k=1}^{\infty} q^{k^2} \cos 2kz$, $\vartheta_4(z, q) \triangleq 1 + 2\Sigma_{k=1}^{\infty}(-1)^k q^{k^2} \cos 2kz$
9.11.5.5	p	$p(z)$	Weierstrass elliptic function ({ $\wp$} is not recommended): $\frac{1}{z^2} + \sum_{\lvert m\rvert + \lvert n\rvert \neq 0}^{-\infty < m,\ n < +\infty} \left\{ \frac{1}{(z-\tilde{\omega}_{mn})^2} - \frac{1}{\tilde{\omega}_{mn}^2} \right\}$, where $\tilde{w}_{mn} \triangleq 2mw_1 + 2nw_2$
9.11.5.6	ζ	$\zeta(z)$	Riemann zeta function: $\sum_{n=1}^{\infty} \frac{1}{n^z}$ $(\mathrm{Re}\ z > 1)$
9.11.5.7	Π	$\Pi(n, z)$	Gauss pi function: $n^z \prod_{k=1}^{n} \frac{k}{k+1}$ $(z \neq -1, -2, \ldots, -n)$ $\lim_{n\to\infty} \Pi(n, z) \triangleq \Pi(z) \equiv \Gamma(z+1)$ (qv 9.11.4.3.1)

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9.12 Probability and Statistics

ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.12.1	E	$E(x)$	expected value of x
9.12.2	P	$P(A)$	unconditional probability that an event, A, occur
9.12.3	P	$P(B\|A)$	conditional probability that an event, B, occur, given that A has occurred
9.12.4	Q	$Q(A)$	unconditional probability that an event, A, *not* occur; $Q(A) \equiv 1 - P(A)$
9.12.5	pdf		probability density function
9.12.6	cdf		cumulative distribution function
9.12.7	irv		independent random variable
9.12.8	d.f.		degrees of freedom
9.12.9	^	$\hat{\theta}$	unbiased estimate of the value of a statistical parameter, θ, from a population
9.12.10	φ	$\varphi(x)$	normal (Gaussian) pdf; with mean, μ, and standard deviation, σ: $\frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$
9.12.11	Φ	$\Phi(x)$	normal (Gaussian) cdf; standard form, i.e., zero mean, unit standard deviation: $\int_{-\infty}^{x} \varphi(t)\,dt \triangleq \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2}\,dt$
9.12.12	χ^2	$\chi^2(x)$	chi-square pdf: $\frac{x^{n/2-1} e^{-x/2}}{2^{n/2}\Gamma(n/2)} (x > 0)$ (qv 9.11.4.3.1) where $n \triangleq$ number of degrees of freedom
9.12.13	μ	μ_m	m^{th} moment about the mean: $\frac{1}{n}\sum_{i=1}^{n} (x_i - \bar{x})^m$, where $n \triangleq$ number of samples of x
9.12.14	α	α_m	standardized μ_m: $\frac{\mu_m}{\sqrt{\mu_2^m}}$; $\alpha_1 = 0$, $\alpha_2 = 1$, $\alpha_3 \triangleq$ skewness, $\alpha_4 \triangleq$ kurtosis
9.12.15	–	$\bar{x}$	sample mean value of x: , $\frac{1}{n}\sum_{i=1}^{n} x_i$, where $n \triangleq$ number of samples of x
9.12.16	~	$\tilde{x}$	population mean value of x: $E(\bar{x})$

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ITEM NUMBER	SIGN OR SYMBOL	APPLICATION	DEFINITION / MEANING
9.12.17	s^2	s_x^2	sample variance of x: $\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2 \equiv \mu_2$, where $n \triangleq$ number of samples of x
9.12.18	σ^2	σ_x^2	population variance of x: $\frac{n}{n-1}E(s_x^2)$, where $n \triangleq$ number of samples of x
9.12.19	s	s_x	sample standard deviation of x: $\sqrt{s_x^2}$
9.12.20	σ	σ_x	population standard deviation of x: $\sqrt{\sigma_x^2}$ $\equiv \sqrt{\frac{n}{2}}\frac{\Gamma(\overline{n-1/2})}{\Gamma(n/2)}E(s_x)$ (qv 9.11.4.4.3.1), where $n \triangleq$ number of samples of x
9.12.21	S	S_y	standard error of estimate of y: $\left[\frac{1}{n}\sum_{i=1}^{n}\{y(x_i)-y_i\}^2\right]^{\frac{1}{2}}$, where $n \triangleq$ number of x–y sample pairs
9.12.22	r^2	$r^2(y)$	coefficient of determination: $1 - S_y^2/s_y^2$
9.12.23	r	r_{xy}	correlation coefficient of x with y: $\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\bar{x}}{s_x}\right)\left(\frac{y_i-\bar{y}}{s_y}\right)$, where $n \triangleq$ number of x–y sample pairs

10. Greek Characters

Because Greek letters traditionally have been used extensively in mathematical works, it is appropriate to provide a guide for their use in mathematical writing. With the advent of electronic computers and their associated software, publications that contain mathematical symbols no longer need to be typeset manually or even with automatic mechanical processes. Computer software for composing mathematical text, including Greek characters, is widely available.[20] Most all computer generated Greek letter fonts contain, at least, both lowercase and uppercase letters that differ from roman fonts. Such a font, in 10 point type, is displayed in the table below. Those letter symbols enclosed in brackets were produced from a roman font.

[20]This Standard was composed using a document preparation system, freely available, known as LATEX, qv [B8], Clause 11.

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LETTER NAME	LETTER SYMBOL		
	UPPERCASE	LOWERCASE	VARIANT
alpha	[A]	α	
beta	[B]	β	
gamma	Γ	γ	
delta	Δ	δ	
epsilon	[E]	∈	ε
zeta	[Z]	ζ	
eta	[H]	η	
theta	Θ	θ	ϑ
iota	[I]	ι	
kappa	[K]	κ	ϰ
lambda	Λ	λ	
mu	[M]	μ	
nu	[N]	ν	
xi	Ξ	ξ	
omicron	[O]	[*o*]	
pi	Π	π	ϖ
rho	[P]	ρ	ϱ
sigma	Σ	σ	ς
tau	[T]	τ	
upsilon	ϒ	υ	
phi	Φ	*ϕ*	φ
chi	[X]	χ	
psi	Ψ	ψ	
omega	Ω	ω	

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