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ASME
PTC 19.1-1998

Test Uncertainty

Instruments and Apparatus

SUPPLEMENT TO ASME PERFORMANCE TEST CODES

The American Society of Mechanical Engineers

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Date of Issuance: December 31, 1998

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FOREWORD

(This Foreword is not a part of ASME PTC 19.1-1998.)

The work on the current edition was begun in 1991. The two-fold objective was to improve the usefulness to the reader as regards clarity, conciseness and technical treatment of an evolving subject matter, as well as harmonization with the ISO "Guide to the Expression for the Uncertainty in Measurement."

This PTC Supplement was approved by the Board on Performance Test Codes on 30 May 1996. It was approved as an American National Standard by the ANSI Board of Standards Review on 20 March 1998.

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NOTICE

All Performance Test Codes **MUST** adhere to the requirements of **PTC 1, GENERAL INSTRUCTIONS.** The following information is based on that document and is included here for emphasis and for the convenience of the user of this Supplement. It is expected that the Code user is fully cognizant of Parts I and III of PTC 1 and has read them prior to applying this Supplement.

ASME Performance Test Codes provide test procedures which yield results of the highest level of accuracy consistent with the best engineering knowledge and practice currently available. They were developed by balanced committees representing all concerned interests. They specify procedures, instrumentation, equipment operating requirements, calculation methods, and uncertainty analysis.

When tests are run in accordance with a Code, the test results themselves, without adjustment for uncertainty, yield the best available indication of the actual performance of the tested equipment. ASME Performance Test Codes do not specify means to compare those results to contractual guarantees. Therefore, it is recommended that the parties to a commercial test agree **before starting the test and preferably before signing the contract** on the method to be used for comparing the test results to the contractual guarantees. It is beyond the scope of any Code to determine or interpret how such comparisons shall be made.

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SECTION 1 — INTRODUCTION

1.1 HARMONIZATION WITH INTERNATIONAL STANDARDS

This Supplement has been substantially rewritten so that it is easier to use. In addition, this Supplement is now in harmony with the International Organization for Standardization (ISO) "Guide to the Expression of Uncertainty in Measurement" (hereafter referred to as the "ISO Guide") [1]. For the practicing engineer, this harmonization means the elimination of such commonly misunderstood terms as bias, precision, bias limit, and precision index. In addition, careful attention is paid to discriminating between errors and the effects of errors.

This Supplement treats recognizable terms such as bias and precision as follows. The term *bias limit,* meaning the estimated limits of the error caused by bias error sources, is now described by the term *systematic uncertainty.* The term *precision* (or *precision index*), meaning the estimate of the magnitude of the error caused by precision error sources, is now described by the term *random uncertainty.* The term for *precision index* has been changed to the *standard deviation of the mean.* While many will prefer to use the terms *bias* and *precision,* using them in strictly unambiguous contexts takes considerable care. This Supplement recommends the terms *systematic uncertainty* and *random uncertainty* as they are less ambiguous.

In addition, this Supplement, while in harmony with the ISO Guide in technology, differs in application. This Supplement emphasizes the effects of uncertainties rather than the source of information about them. The ISO Guide classifies uncertainties by the source of information about them, i.e., are there data to calculate a standard deviation or not? If there are, the ISO Guide calls those uncertainties "Type A," if not, they are called "Type B." This Supplement permits that kind of classification through the vehicle of an optional subscript on each uncertainty term, "A" or "B." However, the recommended and most commonly used classification system for uncertainties in this Supplement is by their effect. That is, if an uncertainty source causes scatter in test results, it is a random uncertainty source and has been caused by random errors. If not, it is a systematic uncertainty and has been caused by systematic errors.

The actual classification of each uncertainty source is left to the analyst. This Supplement merely recommends grouping them as random or systematic with subscripts to identify the origin of information about their magnitudes, if desirable.

The harmonization of this Supplement with the ISO Guide has been achieved by recommending a simplified U_{RSS} uncertainty model which combines the technologies in the ISO Guide and in the earlier version of this Supplement [2]. With this model, detailed in Sections 6 and 7, several simplifying assumptions have been made, namely:

(a) All systematic uncertainty sources are assumed normally distributed and are estimated as 2σ for 95% coverage. If this assumption is not adequate for analysis, more complex methods are handled in Appendix B. Although this assumption is not statistically rigorous, it has been found to work well in many applications.

(b) All random uncertainty sources are estimated as $2S_{\bar{X}}$, which is a 95% confidence estimate of the effect on the average of a particular random uncertainty source.

(c) These uncertainty estimates are grouped as systematic or random and root-sum-squared to obtain the systematic and random uncertainties of a measurement, B and $2S_{\bar{X}}$ for large samples.

(d) These are then root-sum-squared to obtain a 95% confidence uncertainty as follows:

$$U_{95} = [B^2 + (2S_{\bar{X}})^2]^{1/2}$$

This simplified uncertainty model is in harmony with the ISO Guide, which can be shown by rearranging it as

$$U_{95} = 2[(B/2)^2 + (S_{\bar{X}})^2]^{1/2}$$

Here the multiplier 2 is really Student's t_{95} which is taken as 2 for more than 30 degrees of freedom.

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If fewer degrees of freedom are available, the methods for handling that situation are in Appendix B.

This simple approach to uncertainty analysis will meet the needs of most users; details needed for more complex analyses are presented in the Appendices.

Harmonization with the ISO Guide has thus been achieved by using an uncertainty model already developed and used in [2] while preserving historical groupings of elemental uncertainties, both systematic (formerly bias) and random (formerly precision).

1.2 APPLICATIONS

This Supplement is intended to serve as reference to the various other ASME Instruments and Apparatus Supplements (PTC 19 Series) and to the ASME Performance Test Codes in general.

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SECTION 2 — OBJECT AND SCOPE

2.1 OBJECT

The object of this Supplement is to define, describe, and illustrate the various terms and methods used to provide meaningful estimates of measurement uncertainty and effects of individual measurement uncertainties on test results.

Measurement uncertainty analysis is useful because it

(a) identifies dominant sources of error, their effects on a test result, and estimates of their limits, uncertainties;

(b) facilitates communication regarding measurement results;

(c) facilitates the choice of appropriate and cost effective measurement devices and procedures;

(d) reduces the risk of making erroneous decisions; and

(e) demonstrates compliance with agreements.

2.2 SCOPE

The scope of this Supplement is to specify procedures for evaluation of uncertainties in individual test measurements, arising from both random errors and systematic errors, and for the propagation of random and systematic uncertainties into the uncertainty of a test result. The various statistical terms involved are defined in the Nomenclature (para. 3.1) and Glossary (para. 3.2).

The end result of a measurement uncertainty analysis is to provide numerical estimates of systematic uncertainties, random uncertainties, and the combination of these into a total uncertainty with an approximate confidence level.

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SECTION 3 — DEFINITIONS AND DESCRIPTION OF TERMS

3.1 NOMENCLATURE

B = the systematic uncertainty component of a parameter

$$B = \left(\sum_i b_i^2\right)^{1/2}$$

b = the systematic uncertainty associated with an elemental error source

B_R = the systematic uncertainty component of a result

B_{xy} = the covariance of the systematic uncertainties in x and y

B^+, B^- = the upper and lower values of a nonsymmetrical systematic uncertainty

C = the number of coefficients in a regression

M = the number of redundant instruments; also the number of samples

N = the number of measurements or sample points or observations available (the sample size)

P = an independent parameter

R = the result

r = the sample correlation coefficient

S_R = the standard deviation of a result

S_x = the standard deviation of a data sample; an estimate of the standard deviation of the population, σ_x

$$S_X = \sqrt{\sum_{k=1}^{N} \frac{(X_k - \bar{X})^2}{N-1}}$$

where X_k is the kth value of the variable and the degrees of freedom is $N - 1$

S_x^2 = the unbiased estimate of the population variance

$S_{\bar{x}}$ = the estimate of the standard deviation of the mean of N measurements

$$S_{\bar{x}} = \frac{S_x}{\sqrt{N}}$$

SEE = the standard error of estimate of a least squares regression or curve fit. Y_{calc} are values of the regression corresponding to the independent variable, Y_{meas} are values corresponding to the N data points, C represents the number of coefficients of the regression

$$SEE = \left[\frac{\sum_{k=1}^{N} (Y_{meas,k} - Y_{calc,k})^2}{N - C}\right]^{1/2}$$

t = the Student's t value at a specified confidence level with ν degrees of freedom, i.e., $t_{95,\nu}$

tS_R = product of S_R and the appropriate t statistic forms the random component of total uncertainty

U = the total uncertainty

U^+, U^- = the upper and lower values of the nonsymmetrical total uncertainty

X = an individual observation in a data sample

$\bar{X}$ = the sample mean; the average of a set of N individual observations

$$\bar{X} = \frac{1}{N}\sum_{k=1}^{N} X_k$$

β = the (unknown) true systematic error; the fixed or constant component of δ

δ = the (unknown) total error; the difference between a measurement and the true value

ϵ = the (unknown) true random error; the random component of δ

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θ= the absolute sensitivity; the change in the calculated result Y due to an incremental change in a parameter P

$$\theta = \frac{\partial Y}{\partial P}$$

θ'= the relative sensitivity of a result

$$\theta' = \left(\frac{\partial Y}{Y}\Big/\frac{\partial P}{P}\right) = \frac{P}{Y}\left(\frac{\partial Y}{\partial P}\right)$$

μ= the (unknown) true average of the population
ν= the number of degrees of freedom
σ= the (unknown) true standard deviation of the population
σ^2= the (unknown) true variance of the population

3.2 GLOSSARY

accuracy: the closeness of agreement between a measured value and the true value.

calibration: the process of comparing the response of an instrument to a standard instrument over some measurement range.

calibration hierarchy: the chain of calibrations which links or traces a measuring instrument to a primary standard.

confidence level: the probability that the true value falls within the specified limits.

degrees of freedom (ν): a sample of N values is said to have N degrees of freedom, and the first statistic calculated from it also is said to have N degrees of freedom. The degrees of freedom are reduced by one for each previously calculated statistic used to calculate a new statistic.

elemental error: the systematic or random error associated with a single error source.

elemental uncertainty: the systematic or random uncertainty associated with a single uncertainty source.

estimate: a value calculated from a sample which is used to estimate the true value of an entire population. For example, the sample standard deviation, S_x, is an estimate of the population standard deviation, σ.

influence coefficient: see *sensitivity.*

mean ($\bar{X}$): the arithmetic average of N readings.

measurement error (δ): the true, unknown difference between the measured value and the true value.

parameter: a measured quantity, such as temperature, pressure, or stress, used in deriving a result. The definition differs from that used in statistics.

population: the set of all possible measurements of a parameter.

random error (ϵ): sometimes called precision; the true random error which characterizes a member of a set of measurements. ϵ varies in a random, Gaussian-Normal manner, from measurement to measurement.

result (R): a value calculated from a number of parameters.

sample size (N): the number of individual measurements in a sample or used to calculate a mean.

sensitivity: the ratio of the change in a result to a unit change in a parameter.

population standard deviation (σ): a value which quantifies the dispersion of a population.

standard error of estimate (SEE): the measure of dispersion of the dependent variable about a least squares regression or curve.

statistic: any numerical quantity derived from the sample data. $\bar{X}$ and $S_{\bar{x}}$ are statistics.

Student's t: a value used herein to estimate the uncertainty for a given confidence level, obtained from tables entered with degrees of freedom at the chosen confidence level.

systematic error (β): sometimes called bias; the true systematic or fixed error which characterizes every member of any set of measurements from the population. The constant component of the total measurement error, δ.

systematic uncertainty (B): the 95% confidence level estimate of the limits of a true systematic error, β, often determined by judgment.

traceability: the ability to trace the calibration of a measuring device through a chain of calibrations to a primary standard. Traceability does not guarantee accuracy, it only indicates documented calibration hierarchy.

true value: the actual value of the parameter being measured.

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uncertainty (U): $\pm U$ is the interval about the measurement or result that contains the true value for a given confidence level.

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SECTION 4 — FUNDAMENTAL CONCEPTS

4.1 ASSUMPTIONS

The assumptions inherent in measurement uncertainty analysis include the following:

(a) the test process is defined;

(b) the measurement system and test process are controlled;

(c) all appropriate calibration corrections have been applied;

(d) the test objectives are specified; and

(e) the instrument package and data reduction procedures are defined.

For total uncertainty, 95% confidence levels have been used throughout this document in accordance with accepted practice. Other confidence levels may be used, if required.

4.2 MEASUREMENT ERROR

Every measurement has error which results in a difference between the measured value X and the true value. The difference between the measured value and the true value is the total measurement error, δ. Since the true value is unknown, the total error in a measurement cannot be known and therefore only its limits can be estimated. The total measurement error consists of two components: systematic error and random error (see Fig. 4.1). Accurate measurement requires minimizing both systematic and random errors (see Fig. 4.2).

4.2.1 Random Error. Random error, ϵ, is the portion of the total measurement error that varies in repeated measurements of the true value. The total random error in a measurement is usually the sum of the contributions of several elemental random error sources. Elemental random error sources include those which are known and are controlled, those which are negligible and are ignored, and those which are unknown and whose limits must be estimated. Elemental random errors may arise from nonrepeatability in the measurement system, environmental conditions, data reduction techniques, and measurement methods.

4.2.2 Systematic Error. Systematic error, β, is the portion of the total measurement error that remains constant in repeated measurements of the true value. The total systematic error in a measurement is usually the sum of the contributions of several elemental systematic errors. Elemental systematic errors include those which are known and can be calibrated out, those which are negligible and are ignored, and those which are unknown and whose limits must be estimated. Elemental systematic errors may arise from imperfect calibration corrections, data acquisition systems, data reduction techniques, etc. The nonnegligible systematic errors that remain after calibration contribute to the systematic uncertainty of the measurement.

4.3 UNCERTAINTY

Since every observation or measurement is the combination of the true value of the parameter plus the total measurement error, there is an inherent uncertainty in the use of measurements to represent the true value. The total uncertainty in a measurement is the combination of uncertainty due to systematic error and uncertainty due to random error.

4.3.1 Uncertainty Due to Random Error. Since random error introduces variation or scatter in repeated measurements of a parameter, the uncertainty due to random error may be estimated by inspection of the measurement scatter. The population standard deviation, σ, is a measure of the scatter about the true population mean, μ, caused by random error. For a normal distribution (see Fig. 4.3), the interval $\mu \pm 2\sigma$ will include approximately 95% of the population. An estimate of the population standard deviation is the standard deviation of a data sample, S_X, which is determined by

$$S_X = \sqrt{\sum_{k=1}^{N} \frac{(X_k - \bar{X})^2}{N - 1}} \qquad (4.1)$$

where N is the number of measurements made and

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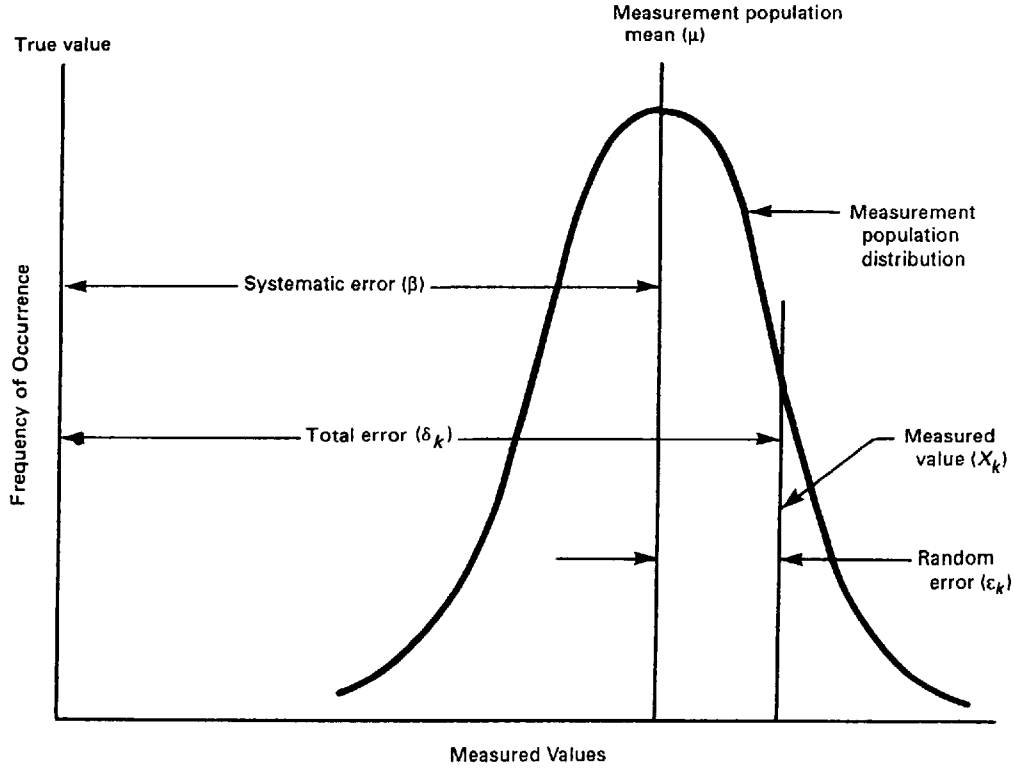

FIG. 4.1 ILLUSTRATION OF MEASUREMENT ERRORS

$\bar{X}$ is the mean of the individual measurements X_k given by

$$\bar{X} = \frac{1}{N}\sum_{k=1}^{N} X_k \qquad (4.2)$$

Averaging multiple measurements and using the mean in lieu of any individual measurement reduces the random uncertainty. This indicates that the mean of several samples is a better measure of the true population mean than one sample. The estimate of the standard deviation of the sample mean, $S_{\bar{X}}$, is given by

$$S_{\bar{X}} = \frac{S_X}{\sqrt{N}} \qquad (4.3)$$

4.3.2 Uncertainty Due to Systematic Error. Since systematic error is constant for repeated measurements of a parameter (via the same technique and constant true value) and since systematic error cannot be quantified absolutely, the uncertainty due to systematic error must be estimated. The systematic uncertainty B is an estimate of the systematic error evaluated at 95% confidence.

The systematic uncertainty estimate is based on engineering judgment and analysis of elemental systematic errors. Every effort must be made to identify and account for all elemental systematic errors. Special tests or data can provide information on the systematic uncertainty in a measurement. The following examples are given in order of preference:

(a) interlaboratory or interfacility tests;

(b) comparison of standards with instruments in the actual test environment;

(c) comparison of independent measurements that depend on different principles or that have been made by independently calibrated instruments; for example, in a gas turbine test, airflow can be measured with an orifice or a bell mouth nozzle, or computed from compressor speed-flow rig data, turbine flow parameters, or jet nozzle calibrations;

(d) special calibrations which perturb a known cause of systematic error through its complete range to determine the range of systematic uncertainty;

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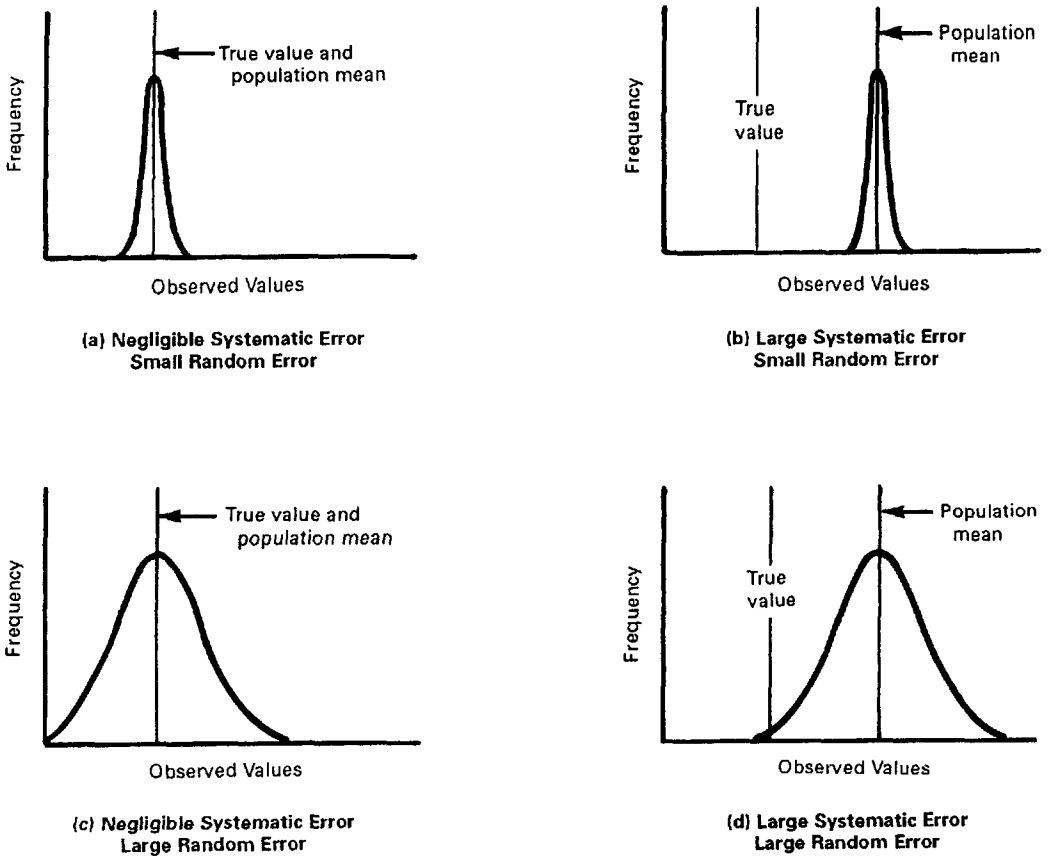

FIG. 4.2 MEASUREMENT ERROR COMPONENTS

(e) reports from instrument manufacturers and other references;

(f) model of the process which is known to generate the error;

(g) engineering judgment or experience.

In most cases, systematic uncertainties are equally likely to be plus or minus about the measurement. That is, typically it is not known if systematic errors are positive or negative and the systematic uncertainty reflects this as $\pm B$. Sometimes, however, the physics of the measurement system provides knowledge of the sign. For a discussion of nonsymmetric systematic uncertainties, see para. 8.2.

In some cases, systematic uncertainties may arise from the same source and are therefore correlated. See para. 8.1 for a detailed discussion.

4.3.3 Total Uncertainty. As mentioned previously, the total uncertainty in a measurement is the combination of uncertainty due to systematic error and uncertainty due to random error. For simplicity of presentation, a single value is often preferred to express a reasonable limit for the total measurement error. Since the true value is unknown, it is convenient to define the limit referenced to the measured value. Uncertainty, U, is defined as an interval, about the measured value, that has a preassigned probability of containing the true value. Therefore, the interval

$$\overline{X} \pm U \tag{4.4}$$

represents a band about the measurement mean, $\overline{X}$, within which the true value is expected to lie at a given level of confidence (see Fig. 6.2).

The uncertainty interval for 95% confidence, U_{95}, is calculated by

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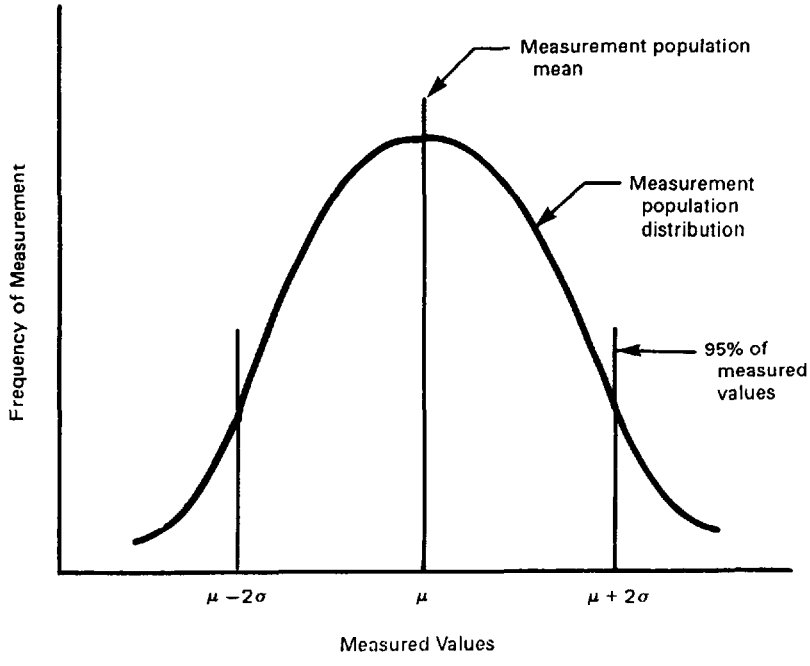

FIG. 4.3 DISTRIBUTION OF MEASURED VALUES (NORMAL DISTRIBUTION)

$$U_{95} = 2\sqrt{\left(\frac{B}{2}\right)^2 + (S_{\bar{X}})^2} \qquad (4.5)$$

where B is the 95% confidence estimate of systematic uncertainty and $S_{\bar{X}}$ is the standard deviation of the mean.

Several assumptions are required for this simple equation. These include:

(a) a 95% confidence interval is appropriate;

(b) the degrees of freedom is sufficiently large such that $t = 2$ can be used for the multiplier; and

(c) the systematic uncertainty component is from a distribution of possible values that is normally distributed.

Treatment of uncertainty intervals with alternative confidence levels and alternate uncertainty equations is addressed in Appendix B.

4.4 PRETEST ANALYSIS AND CORRECTIVE ACTION

It is recommended than an uncertainty analysis be done before a test or experiment. This procedure allows corrective action to be taken prior to the test to reduce uncertainties.

The pretest uncertainty analysis is based on data and information that exist before the test, such as calibration histories, previous tests with similar instrumentation, prior measurement uncertainty analysis, expert opinions, and, if necessary, special tests. With complex tests, there often are alternatives to evaluate prior to the test such as different test designs, instrumentation arrangements, alternative calculation procedures, and so on. Pretest analysis can identify the most accurate test method.

Corrective action resulting from this pretest analysis may include:

(a) improvements to all calibrations of instrumentation if the systematic uncertainties are unacceptable;

(b) selection of a different measurement method to obtain the parameter of interest; and

(c) repeated testing, or increased sample sizes, or both if the random uncertainties are unacceptable.

Cost and time may dictate the choice of the corrective action. If the corrective action identified cannot be taken, there may be a high risk that test objectives will not be met because of the large uncertainty interval, and cancellation of the test should be a consideration.

4.5 POSTTEST ANALYSIS — PHILOSOPHY

4.5.1 Reported Results. The results from a test can normally be presented in terms of an average, $\bar{X}$, with an uncertainty estimate. This can be written as

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TABLE 4.1
DISCHARGE PRESSURE TEST DATA WITH $N = 37$ (EXAMPLE 4.5-1)

	Equation	kPa	psi
Systematic uncertainty, B	. . .	4.10	0.60
Random uncertainty, $2S_{\bar{x}}$	. . .	5.52	0.80
Mean, $\bar{X}$	4.2	758.5	110.00
Sample standard deviation, S_x	4.1	16.77	2.43
Standard deviation of the mean, $S_{\bar{x}}$	4.3	2.76	0.40
Total uncertainty, U	4.5	6.88	1.00

$$\bar{X} \pm U \text{ (95\% confidence)} \qquad (4.6)$$

which implies that, under the conditions that occurred during a single test or a series of tests, the average is $\bar{X}$, and an estimate of the possible error associated with the average $\bar{X}$ at 95% confidence is U. If the test was to determine the supply pressure and efficiency for a particular pump, the test report might read:

> Pump number 6 was tested on 23 July 1993 using company procedure XYZ. For the specified flow of 0.157 m^3/s (2500 gpm) at 20°C, the discharge pressure was 758 ±7 kPa or 110 ± 1 psi (95% confidence). The estimated efficiency of the pump during the testing was 0.812 ± 0.009 (95% confidence).

This test was conducted at a single flow rate and had two results: (1) discharge pressure and (2) efficiency. It must be assumed that the specified procedure presents an acceptable range of flow and temperature for the testing. The statement above implies that the average discharge pressure during the test was 758 kPa, and the measurement uncertainty associated with this pressure measurement was 7 kPa. The pressure would be considered a measurement, and the estimate of the uncertainty would be generated as presented in Section 6. The statement does not provide information on whether the discharge pressure varied appreciably during the test or if the discharge pressure was acceptable.

4.5.2 Posttest Reviews. A strict mathematical review of test data is normally not adequate. Not only is it necessary to algebraically demonstrate conformance to a requirement (e.g., the measurement uncertainty was below that required by the test program), the basic assumptions of para. 4.1 must be checked. Plotting of the data such as shown in Fig. 4.4 is one method to evaluate compliance with the assumptions in para. 4.1. Example 4.5-1 discusses some of the basic checks which could be made with any data set.

Example 4.5-1: Table 4.1 presents the test data for the pump that was described earlier. For this test, the pump was operated for 3 hr and data were collected every 5 min. The systematic uncertainty, B, in Table 4.1 has been estimated based on the calibration data and previous test experience. The remaining quantities have been computed from the raw test data.

The test data presented in Fig. 4.4 are typical of a set of test data; while the measured reading varies with time, there is no observable pattern of behavior. If the data were clumped such that most of the early data lay above the mean while the later data lay below the mean, this would suggest that the test arrangement was not at steady-state. This would indicate that the process under test was not controlled [assumption (b), para. 4.1]. Also, if the data appeared cyclic with time, the process under test may not be controlled.

A final check for any data set is performed by comparison to a known condition. Do the test data meet expectations? For performance testing, the behavior of different classes of equipment is well established. Performance which deviates from an established norm indicates a need for additional evaluation.

Examples of why deviations might exist are:

(a) the tested device does not perform as expected;

(b) the test facility has changed; and

(c) an uncontrolled test variable exists.

If the pretest and posttest uncertainties do not compare within expectations, it is possible that the assumptions of para. 4.1 have been violated. For a further discussion of these issues, a text on experimental methods should be consulted.

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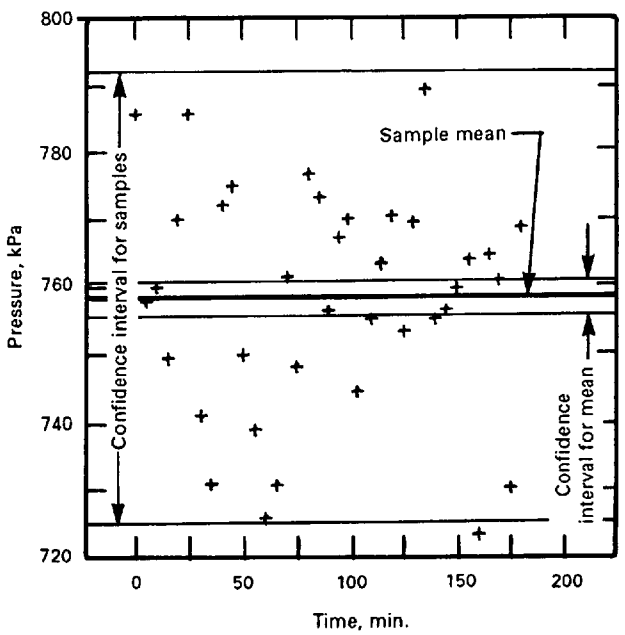

FIG. 4.4 TEST DATA (EXAMPLE 4.5-1)

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SECTION 5 — DEFINING THE MEASUREMENT PROCESS

The first step in an uncertainty analysis is to clearly define the measurement process. This simple step, often overlooked, is essential to successfully develop and apply the uncertainty information. Consideration must be given to the selection of the appropriate "true value" of the measurement and the time interval for classifying errors as systematic or random.

5.1 SELECTION OF THE APPROPRIATE "TRUE VALUE"

Depending on the user's perspective, several "true values" may exist simultaneously in a measurement process. For example, when analyzing a thermocouple measurement in a gas stream, several starting points, or "true values," can be selected. The starting point for the analysis could begin with the "true value" defined as the metal temperature of the thermocouple junction, the gas stagnation temperature at the thermocouple point location if the probe was not installed, or the mass flow weighted average of the gas temperature at the plane of the instrumentation. Any of these possible "true values" may be appropriate. The selection of the "true value" for the uncertainty analysis must be consistent with the goal of the measurement [3].

5.2 TIME INTERVAL

Since measurement error sources are classified by their effect, the time interval for the measurement process must also be clearly stated. Errors that may be fixed over a short time period may be variable over a longer time period. For example, the calibration corrections, which may be assumed to be fixed over the life of the calibration interval, can be considered variable if the process consists of a time interval encompassing several recalibrations. The time interval must be clearly specified to classify an error as either systematic or random.

The time interval used to classify an error as either systematic or random is usually, but not always, the same as the test duration. When comparing results between various laboratories, it might be appropriate to classify a quantifiable error source that has a time period much larger than the duration of the test as random even though it was constant for the duration of the test.

5.3 COMPARATIVE VERSUS ABSOLUTE TESTING

The objective of a comparative test (also known as a back-to-back test) is to determine, with the smallest measurement uncertainty possible, the net effect of a design change. The first test is run with the standard or baseline configuration. The second test is then run in the same facility with the design change and hopefully with instruments, setups, and calibrations identical to those used in the first test. The difference between the results of these tests is an indication of the effect of the design change. Because of the effects of correlated uncertainties (see Section 8), the total uncertainty of the difference will usually be less than the uncertainty of each separate test. An example of back-to-back uncertainty analysis is shown in para. 8.1.

5.4 MEASUREMENT UNCERTAINTY SOURCES

Sources introducing uncertainty in a measurement process can be divided arbitrarily into categories, e.g.,

(a) calibration uncertainty
(b) data acquisition uncertainty
(c) data reduction uncertainty
(d) uncertainty due to methods
(e) others

While often helpful, dividing uncertainty sources by categories is not necessary for a correct uncertainty analysis.

To decide if a given elemental uncertainty source contributes to the systematic or random component of the total uncertainty, the effect of the error on the measurement must be determined. If the nature of an elemental error is fixed over the duration of the defined measurement process, then a 95%

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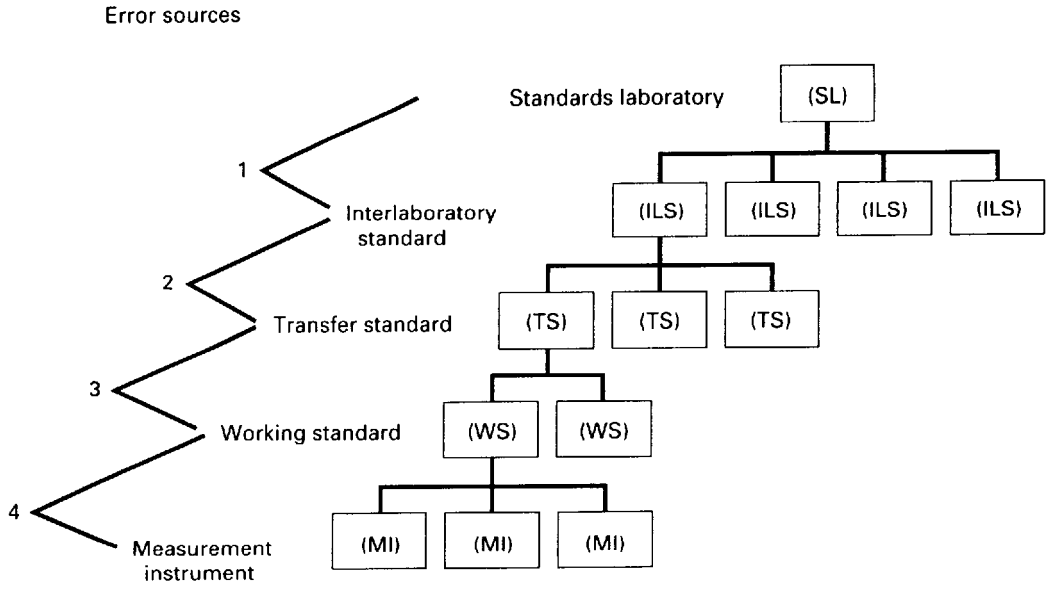

FIG. 5.1 GENERIC MEASUREMENT CALIBRATION HIERARCHY

confidence estimate of the error is classified as a systematic uncertainty. If the error source tends to cause scatter in repeated observations of the defined measurement process, then the source contributes to the random uncertainty. The methods used to estimate uncertainty, as presented in this document, may be used to estimate each of the five above uncertainties.

Secondarily, it may also be beneficial to characterize the elemental uncertainty sources by the manner in which they are estimated. Those which are evaluated by statistical methods may be classified as Type A while those which are evaluated by other means may be classified as Type B. Depending on the selection of the defined measurement process, there may be no simple correspondence between random or systematic and Type A or Type B.

5.4.1 Calibration Uncertainty. Each measurement instrument may introduce random and systematic uncertainties. The main purpose of the calibration process is to eliminate large, known systematic errors and thus reduce the measurement uncertainty to some "acceptable" level. Having decided on the "acceptable" level, the calibration process achieves that goal by exchanging the large systematic uncertainty of an uncalibrated or poorly calibrated instrument for the smaller combination of systematic uncertainties of the standard instrument and the random uncertainties of the comparison.

Calibrations are also used to provide traceability to known reference standards or physical constants, or both. Requirements of military and commercial contracts have led to the establishment of extensive hierarchies of standards laboratories. In some countries, a national standards laboratory is at the apex of these hierarchies, providing the ultimate reference for every standards laboratory. Each additional level in the calibration hierarchy adds uncertainty in the measurement process (see Fig. 5.1).

5.4.2 Data Acquisition Uncertainty. Uncertainty in data acquisition systems can arise from errors in the signal conditioning, the sensors, the recording devices, etc. The best method to minimize the effects of many of these uncertainty sources is to perform overall system calibrations. By comparing known input values with their measured results, estimates of the data acquisition system uncertainty can be obtained. However, it is not always possible to do this. In these cases, it is necessary to evaluate each of the elemental uncertainties and to combine them to predict the overall uncertainty.

5.4.3 Data Reduction Uncertainty. Computations on raw data are done to produce output (data) in engineering units. Typical uncertainty sources in this

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category stem from curve fits and computational resolution. With the recent advances in computer systems, the computational resolution uncertainty sources are often negligible; however, curve fit error uncertainty can be significant.

5.4.4 Uncertainty Due to Methods. Uncertainties due to methods are defined as those additional uncertainty sources that originate from the techniques or methods inherent in the measurement process. These uncertainty sources, beyond those contained in calibration, data acquisition, and data reduction, may significantly affect the uncertainty of the final results. Some common examples include:

(a) uncertainty in the assumptions or constants contained in the calculation routines;

(b) uncertainty due to intrusive disturbance effects caused by installed instrumentation;

(c) spatial or profile uncertainty in the conversion from discrete point measurements to station averages;

(d) environmental effects on probes such as conduction, convection, and radiation;

(e) uncertainty due to instability, nonrepeatability, and hysteresis of the test process.

These additional uncertainties may be of either a systematic or random nature depending on their effect on the measurement.

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SECTION 6 — UNCERTAINTY OF A MEASUREMENT

6.1 DETERMINATION OF SAMPLE STANDARD DEVIATION AND DEGREES OF FREEDOM

6.1.1 General Case. For $\overline{X}$ that is determined as the average of N readings, the appropriate sample standard deviation of the mean ($S_{\overline{X}}$) is given by Eq. (4.3).

In a sample, the degrees of freedom is the sample size (N). When a statistic is calculated from the sample, the degrees of freedom associated with the statistic is reduced by one for every estimated parameter used in calculating the statistic. For example, from a sample of size N, $\overline{X}$ is calculated by Eq. (4.2) and has N degrees of freedom. The sample standard deviation (S_X) and the sample standard deviation of the mean ($S_{\overline{X}}$) are calculated from Eqs. (4.1) and (4.3), respectively, and each has $N - 1$ degrees of freedom (ν)

$$\nu = N - 1 \tag{6.1}$$

because $\overline{X}$ (based on the same sample of data) is used in the calculation of both standard deviations.

6.1.2 Using Previous Values With Small Samples. In some test situations, the measurement of a variable may be only a single reading or a single average of readings taken over a short time frame, as with a computer-based data acquisition system. In the latter case, the time frame over which the readings are taken may be on the order of milliseconds or less while the random variations in the process may be on the order of seconds, or minutes, or even days. This "averaged" reading should then be handled in the same manner as a single reading.

Information about the possible variations in a single reading measurement must be obtained from previous readings of the variable taken over the time frame and conditions which cover the variations in the variable. For example, taking multiple samples of data as a function of time while holding all other conditions constant would identify the random variation associated with the measurement system and the unsteadiness of the test condition. If the sample standard deviation of the variable being measured is also expected to be representative of other possible random variations in the measurement, e.g., repeatability of test conditions, variation in test configuration, etc., then these additional error sources will have to be varied while the multiple data samples are taken to determine the standard deviation.

Another situation where previous readings of a variable would be useful is when a small sample size (N) is used to calculate the mean value ($\overline{X}$) of a measurement. If a much larger set of previous readings for the same test conditions is available, then it could be used to calculate a more appropriate standard deviation for the measurement [4]. Typically, these larger data sets are taken in the early phases of an experimental program. Once the random variation of the test variables is understood, then this information can be used to streamline the test procedure by reducing the number of readings taken in the later measurements.

When previous readings (X_{P_k}) are known for the quantity being measured, the sample standard deviation for the variable can be calculated from these N_P previous readings as

$$S_X = \left[\frac{1}{N_P - 1}\sum_{k=1}^{N_P}(X_{P_k} - \overline{X}_P)^2\right]^{1/2} \tag{6.2}$$

where

$$\overline{X}_P = \frac{1}{N_P}\sum_{k=1}^{N_P} X_{P_k} \tag{6.3}$$

The appropriate standard deviation of the mean for the current measurement ($\overline{X}$) is then

$$S_{\overline{X}} = \frac{S_X}{\sqrt{N}} \tag{6.4}$$

where N is the number of current readings averaged to determine $\overline{X}$. The number of degrees of freedom for this standard deviation of the mean $S_{\overline{X}}$ is

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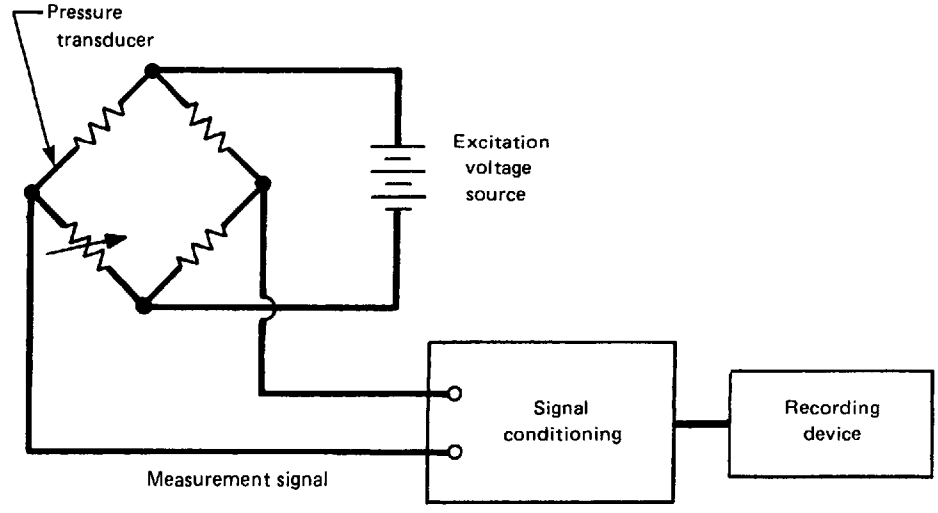

FIG. 6.1 PRESSURE DATA ACQUISITION SYSTEM

$$\nu = N_P - 1 \tag{6.5}$$

The case where a measurement is only a single reading is handled above with $N = 1$.

6.1.3 Using Elemental Random Error Sources. Another method of determining the standard deviation of the mean for a measurement is from information about the elemental random error sources in the entire measurement process. As an example, consider the pressure data acquisition system shown in Fig. 6.1. Elemental random errors for this system can arise from several sources such as the excitation voltage, the signal conditioning process, the recording device, the pressure transducer, probe errors, and environmental effects.

The standard deviation for the measurement mean is the root-sum-square of the elemental standard deviations of the mean from all sources divided by the square root of the number of current readings (N) averaged to determine $\overline{X}$

$$S_{\overline{X}} = \frac{1}{\sqrt{N}}\left[\sum_{i=1}^{K} (S_{\overline{X}})_i^2\right]^{1/2} \tag{6.6}$$

where K is the total number of random error (or uncertainty) sources.

Each of the elemental standard deviations of the mean $(S_{\overline{X}})_i$ is calculated using the methods described in para. 6.1.1 or 6.1.2 depending upon which is appropriate. If in each of the N readings of the measurement X the output of an elemental component is averaged N_i times to obtain $\overline{X}_i$, then the method in para. 6.1.1 would be used. If instead previous information is used to obtain $(S_{\overline{X}})_i$, then the method in para. 6.1.2 would apply.

The degrees of freedom for the measurement standard deviation of the mean $(S_{\overline{X}})$ is dependent on the information used to determine each of the elemental standard deviations of the mean and is calculated as

$$\nu = \frac{\left(\sum_{i=1}^{K} (S_{\overline{X}})_i^2\right)^2}{\sum_{i=1}^{K} \frac{(S_{\overline{X}})_i^4}{\nu_i}} \tag{6.7}$$

where ν_i is the appropriate degrees of freedom for $(S_{\overline{X}})_i$ and is obtained from Eq. (6.1) or (6.5) as appropriate. When all error sources have large sample sizes, the calculation of ν is unnecessary. However, for small samples, when combining elemental standard deviations of the mean by the root-sum-square method [see Eq. (6.6)], the degrees of freedom (ν) associated with the combined standard deviation is calculated using the Welch-Satterthwaite formula [5] above [Eq. (6.7)].

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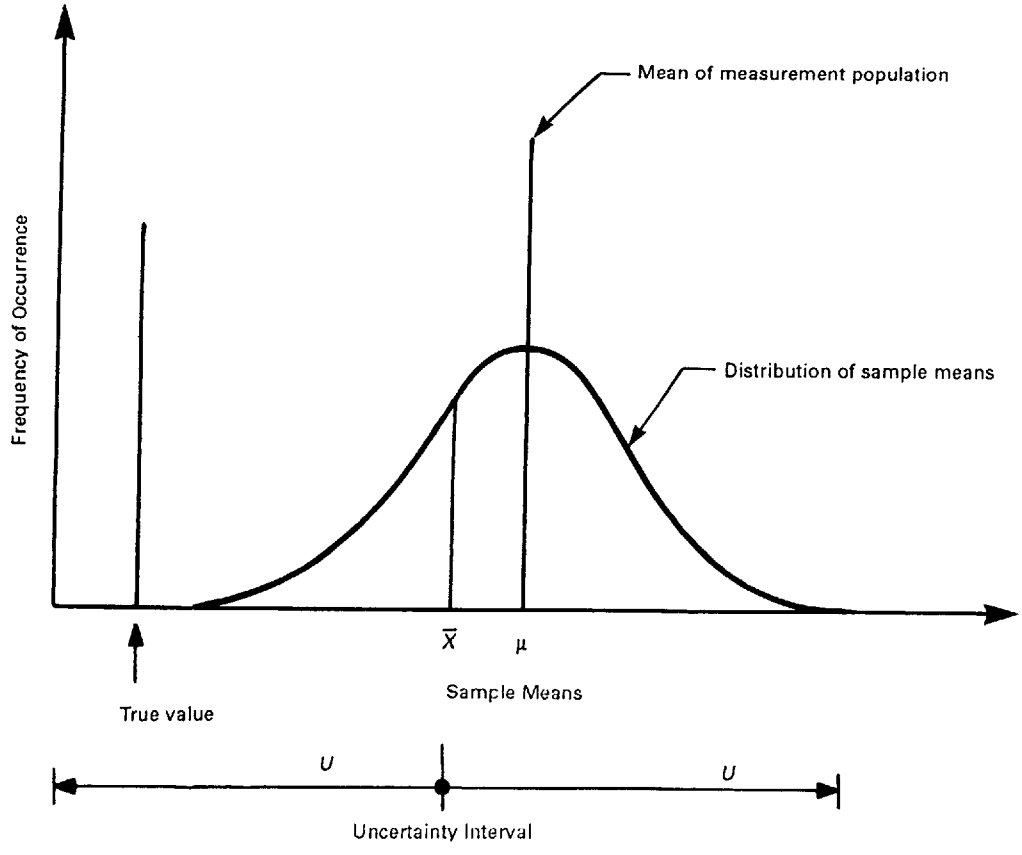

FIG. 6.2 UNCERTAINTY INTERVAL

6.2 COMBINING ELEMENTAL SYSTEMATIC UNCERTAINTIES

The systematic uncertainty (B) of a measurement was defined in para. 4.3.2 as the 95% confidence level estimate of the limits of the true systematic error (β). The systematic uncertainty of the measurement is the root-sum-square of the elemental systematic uncertainties (b_i) for all sources

$$B = \left[\sum_{i=1}^{K} b_i^2\right]^{1/2} \qquad (6.8)$$

where K is the total number of systematic error sources. For each error source in the measurement, the elemental systematic uncertainty must be estimated from the best available information. If any of the elemental systematic uncertainties are nonsymmetrical, then the method given in Section 8.2 should be used to determine the systematic uncertainty of the measurement.

There can be many sources of systematic error in a measurement, such as the calibration process, instrument systematic errors, transducer errors, and fixed errors of method. Also environmental effects, such as radiation effects in a temperature measurement, can cause systematic errors of method. There will usually be some elemental systematic uncertainties that will be dominant. Because of the resulting effect of combining the elemental uncertainties in a root-sum-square manner, the larger or dominant ones will control the uncertainty in the measurement; however, one should be very careful to identity all sources of fixed error in the measurement.

6.3 TOTAL UNCERTAINTY OF A MEASUREMENT

For simplicity of presentation, a single value is often preferred to express the estimate of the error between the mean value ($\bar{X}$) and the true value,

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with a defined confidence. Since the true value is unknown, it is convenient to define a limit referenced to the measured value. Uncertainty (U) is defined as an interval, about the measured value, that has a preassigned probability of containing the true value. The interval

$$\overline{X} \pm U \tag{6.9}$$

represents a band about $\overline{X}$ within which the true value is expected to lie with a given level of confidence (see Fig. 6.2). The uncertainty interval is composed of both the systematic and random uncertainty components.

The general form of the expression for determining the uncertainty of a measurement is the root-sum-square of the systematic and random standard deviations of the measurement times the appropriate Student's t value for a specified confidence level (see Appendix B). Depending on the application, various confidence levels may be appropriate. A Student's t is chosen on the basis of the desired level of confidence and degrees of freedom. A t value of 1.96 (usually taken as 2) corresponds to large degrees of freedom and defines an interval with a level of confidence of approximately 95%. For other confidence levels see Appendix B.

The systematic uncertainty (B) for a specific measurement is the estimate of the limits of the fixed error in that measurement. The true systematic error (β) is unknown, but B is estimated so that it represents the limits of a 95% confidence interval for the possible β values. For the simple uncertainty expression given below, it is assumed that these possible β values are from a Gaussian distribution and that enough information is known about the possible systematic errors so that the degrees of freedom of the distribution is large. Then the systematic uncertainty (B) can be considered to be two times the standard deviation of the systematic error distribution. It should be noted that while B is a 95% estimate of the distribution of possible bias errors in a measurement, the bias error that is present in a specific measurement is a fixed single value of β.

The uncertainty interval for 95% confidence ($t = 2$) is calculated by

$$U_{95} = 2\sqrt{\left(\frac{B}{2}\right)^2 + (S_{\overline{X}})^2} \tag{6.10}$$

where B is the 95% confidence estimate of systematic uncertainty [Eq. (6.8)], $S_{\overline{X}}$ is the standard deviation of the mean [Eq. (4.3), (6.4), or (6.6) as appropriate], and U_{95} is the uncertainty interval for 95% confidence.

Several assumptions are required for this simple equation. These include:

(a) a 95% confidence interval is appropriate;

(b) the degrees of freedom is sufficiently large such that $t = 2$ can be used for the multiplier;

(c) the systematic uncertainty component is from a distribution of possible values that is symmetric, normally distributed, and has large degrees of freedom.

Treatment of uncertainty intervals with alternate confidence levels and alternate uncertainty equations is addressed in Appendix B.

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SECTION 7 — UNCERTAINTY OF A RESULT

7.1 PROPAGATION OF MEASUREMENT UNCERTAINTIES INTO A RESULT

Results are not measured directly; more basic parameters such as temperature and pressure are measured and the required result is calculated as a function of measurements of these parameters. Uncertainties in measurements of the parameters are propagated to the result through the functional relationship between the result and the parameters. The effect of the propagation can be approximated by the Taylor series method (see Appendix C).

To obtain the uncertainty of a derived result, such as turbine efficiency, based on a series of measured parameters, such as temperature and pressure, it is necessary to express the result (R) in terms of the average values of the independent parameters ($\overline{P}_i$) that enter into the result. That is,

$$R = f(\overline{P}_1, \overline{P}_2, \ldots, \overline{P}_J) \tag{7.1}$$

where the subscript J signifies the total number of parameters involved in R, and the average values of the independent parameters are obtained as

$$\overline{P}_i = \frac{1}{N_i} \sum_{k=1}^{N_i} P_{i_k} \tag{7.2}$$

where N_i is the number of repeated measurements of P_i. N_i will be 1 for a single reading.

In some experimental situations, a set of parameters ($\overline{P}_i$) is measured and a single result (R) is calculated. This case is called a single test result. In this case, some of the parameters may be single measurements and others may be means of N_i measurements. N_i can be different for each $\overline{P}_i$.

When, in an experiment, multiple test results can be run at the same condition and each parameter is measured during each test, then these multiple sets of data can be used to determine multiple results. The reported result will usually be the mean of the results ($\overline{R}$). This case is called a multiple test result in the discussions of uncertainty given in the following paragraphs.

7.2 SENSITIVITY

It is convenient to introduce the concept of the sensitivity of a result to a unit change in each of its parameters. Sensitivity is the ratio of the change in a result to a unit change in a parameter. The sensitivity coefficient of each parameter is obtained in one of two ways.

7.2.1 Analytically. When there is a known mathematical relationship between the result (R) and its parameters ($\overline{P}_1, \overline{P}_2, \ldots, \overline{P}_J$), then the absolute (dimensional) sensitivity coefficient (θ_i) of the parameter ($\overline{P}_i$) may be obtained by partial differentiation.

Thus if $R = f(\overline{P}_1, \overline{P}_2, \ldots, \overline{P}_J)$, then

$$\theta_i = \frac{\partial R}{\partial \overline{P}_i} \tag{7.3}$$

Analogously, the relative (nondimensional) sensitivity coefficient (θ_i') is

$$\theta_i' = \frac{\dfrac{\partial R}{\dfrac{R}{\partial \overline{P}_i}}}{\overline{P}_i} = \frac{\overline{P}_i}{R}\left(\frac{\partial R}{\partial \overline{P}_i}\right) \tag{7.4}$$

7.2.2 Numerically. Finite increments in a parameter also may be used to evaluate sensitivity using the data reduction calculation procedure.

In this case, θ_i is given by

$$\theta_i = \frac{\Delta R}{\Delta \overline{P}_i} \tag{7.5}$$

and θ_i' by

$$\theta_i' = \frac{\dfrac{\Delta R}{\dfrac{R}{\Delta \overline{P}_i}}}{\overline{P}_i} = \frac{\overline{P}_i}{R}\left(\frac{\Delta R}{\Delta \overline{P}_i}\right) \tag{7.6}$$

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The result is calculated using $\overline{P}_i$ to obtain R and then recalculated using $(\overline{P}_i + \Delta\overline{P}_i)$ to obtain $(R + \Delta R)$ [6].

To best approximate the sensitivity that would be obtained analytically, the value of $\Delta\overline{P}_i$ used should be as small as practical (i.e., large enough to keep round-off errors from influencing the calculations).

7.3 RANDOM STANDARD DEVIATION OF A RESULT

7.3.1 Single Test. The absolute standard deviation of a single test result may be determined from the propagation equation (see Appendix C) as

$$S_R = \left[\sum_{i=1}^{J} (\theta_i S_{\overline{P}_i})^2\right]^{1/2} \tag{7.7}$$

The relative standard deviation of a result is

$$\frac{S_R}{R} = \left[\sum_{i=1}^{J} \left(\theta_i' \frac{S_{\overline{P}_i}}{\overline{P}_i}\right)^2\right]^{1/2} \tag{7.8}$$

The symbols θ_i and θ_i' are the absolute and relative sensitivity coefficients, respectively, of Eqs. (7.3) or (7.5) and (7.4) or (7.6), and $S_{\overline{P}_i}$ is the standard deviation of the mean of the measured parameter average $(\overline{P}_i)$, determined according to the methods presented in para. 6.1.

7.3.2 Multiple Test. When more than one test is conducted with the same instrument package (i.e., repeated tests), the uncertainty of the average test result will be less than that for one test because of the reduction in the random uncertainty of the average. However, systematic uncertainty will remain the same as for a single test.

Careful consideration should be given to designing the test series to average as many causes of variation as possible within cost constraints. The test design should be tailored to the specific situation [7]. For example, if experience indicates that time-to-time and rig-to-rig variations are significant, a test design that averages multiple test results on one rig or for only one day may produce optimistic random uncertainty estimates compared to testing several rigs, each monitored several times over a period of several days. The list of test variation causes are many and may include the above plus environmental and test crew variations. Historic data are invaluable for studying these effects. A statistical technique called analysis of variance (ANOVA) is useful for partitioning the total variance by cause [5]. If the pretest uncertainty analysis (see para. 4.4) identifies unacceptably large error sources, special tests to measure the effects of these sources should be considered.

The average result from more than one test is given by

$$\overline{R} = \frac{\sum_{i=1}^{M} R_i}{M} \tag{7.9}$$

where M signifies the number of tests available.

Following Eq. (4.1), the estimate of the standard deviation of the distribution of the results is

$$S_R = \left[\frac{\sum_{i=1}^{M} (R_i - \overline{R})^2}{M-1}\right]^{1/2} \tag{7.10}$$

where S_R includes random errors within tests and variation between tests. The degrees of freedom associated with S_R is determined by $\nu_R = M - 1$.

The sample standard deviation of the mean result from multiple tests is

$$S_{\overline{R}} = \frac{S_R}{\sqrt{M}} \tag{7.11}$$

This standard deviation of the mean also has $\nu_{\overline{R}} = M - 1$ degrees of freedom.

7.4 SYSTEMATIC UNCERTAINTY OF A RESULT

In para. 6.3, the assumption was made that if the possible systematic errors for the measured parameters were from Gaussian distributions, and that if enough information were known about the possible systematic errors so that the degrees of freedom of the distributions were large, then the systematic uncertainty for each parameter would be essentially an estimate of two times the standard deviation of the systematic error distribution. Using these assumptions, the propagation equation in Appendix C can be used to determine the absolute systematic uncertainty of a result as

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$$B_R = \left[\sum_{i=1}^{J} (\theta_i B_{\bar{P}_i})^2\right]^{1/2} \quad (7.12)$$

The relative systematic uncertainty of a result is

$$\frac{B_R}{R} = \left[\sum_{i=1}^{J} \left(\theta_i' \frac{B_{\bar{P}_i}}{\bar{P}_i}\right)^2\right]^{1/2} \quad (7.13)$$

The symbol $B_{\bar{P}_i}$ is the systematic uncertainty of the measured parameter (see para. 6.2).

7.5 UNCERTAINTY OF A RESULT

The general form of the expression for determining the uncertainty of a result is the root-sum-square of the systematic and random standard deviations of the result times the appropriate Student's t value for a specified confidence level (see Appendix B). The following simple expression for the uncertainty of a result applies in many cases:

$$U_{R_{95}} = 2\left[\left(\frac{B_R}{2}\right)^2 + (S_R)^2\right]^{1/2} \quad (7.14)$$

where B_R is obtained from Eq. (7.12) and S_R is obtained from either Eq. (7.7) for a single test result or from Eq. (7.11) for a multiple test result.

The assumptions required to use Eq. (7.14) are that:

(a) a 95% confidence interval is appropriate;

(b) the degrees of freedom is sufficiently large so that $t = 2$ can be used for the multiplier (see Appendix B for the method to calculate the degrees of freedom of the result);

(c) the systematic uncertainty of the result is from a distribution of possible values that is symmetric and normally distributed with degrees of freedom large enough so that the standard deviation is approximated by $B_R/2$.

The interval within which the true result should lie with 95% confidence is

$$R \pm U_{R_{95}} \quad (7.15)$$

The special cases of correlated systematic uncertainties and nonsymmetric systematic uncertainties are covered in paras. 8.1 and 8.2 respectively. Treatment of uncertainty intervals with alternate confidence levels and alternate uncertainty equations is addressed in Appendix B.

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SECTION 8 — ADDITIONAL UNCERTAINTY CONSIDERATIONS

8.1 CORRELATED SYSTEMATIC UNCERTAINTIES

The expressions for the systematic uncertainty of the result in para. 7.4 [Eqs. (7.12) and (7.13)] assume that the systematic uncertainties of the measured parameters are all independent of each other. There are many situations where systematic errors for some of the parameters in a result are not independent. Examples would include using the same apparatus to measure different parameters or calibrating different parameters against the same standard. In these cases, some of the systematic errors are said to be correlated and these nonindependent errors must be considered in the determination of the systematic uncertainty of the result [8].

Consider an example where the result (R) is determined from three parameters ($\overline{X}_1, \overline{X}_2, \overline{X}_3$) which have correlated systematic errors. The result is calculated as

$$R = f(\overline{X}_1, \overline{X}_2, \overline{X}_3) \tag{8.1}$$

and the absolute systematic uncertainty of the result is given as

$$\begin{aligned} B_R = \big[&(\theta_1 B_{\overline{X}_1})^2 + (\theta_2 B_{\overline{X}_2})^2 + (\theta_3 B_{\overline{X}_3})^2 \\ &+ 2\theta_1\theta_2 B_{\overline{X}_1\overline{X}_2} + 2\theta_1\theta_3 B_{\overline{X}_1\overline{X}_3} \\ &+ 2\theta_2\theta_3 B_{\overline{X}_2\overline{X}_3}\big]^{1/2} \end{aligned} \tag{8.2}$$

The first three terms under the square root in Eq. (8.2) are the same as those obtained by using Eq. (7.12), and the last three terms are those that account for the correlation among the systematic uncertainties in $\overline{X}_1$, $\overline{X}_2$, and $\overline{X}_3$. The terms $B_{\overline{X}_i\overline{X}_k}$ are the estimates of the covariances of the systematic errors in $\overline{X}_i$ and $\overline{X}_k$ (see Appendix B). These terms must be included when systematic uncertainties for separate parameters, $\overline{X}_i$ and $\overline{X}_k$, are from the same source making them correlated. The units of the correlation terms, $B_{\overline{X}_i\overline{X}_k}$, are the product of the units of $\overline{X}_i$ and $\overline{X}_k$.

The covariance terms in Eq. (8.2) must be properly interpreted. Each $B_{\overline{X}_i\overline{X}_k}$ term represents the sum of the products of the portions of $B_{\overline{X}_i}$ and $B_{\overline{X}_k}$ that arise from the same source and are therefore perfectly correlated [9]. For instance, if elemental systematic uncertainties 1 and 2 for parameters 2 and 3 were from a common source, then $B_{\overline{X}_2\overline{X}_3}$ would be determined as

$$B_{\overline{X}_2\overline{X}_3} = (b_{\overline{X}_2})_1 (b_{\overline{X}_3})_1 + (b_{\overline{X}_2})_2 (b_{\overline{X}_3})_2 \tag{8.3}$$

The example in Eq. (8.2) can be extended to any number of parameters by including a term $2\theta_i\theta_k B_{\overline{X}_i\overline{X}_k}$ for each pair of parameters with correlated systematic uncertainties.

Example 8.1: The use of back-to-back tests is an excellent method to reduce the systematic uncertainty when comparing two or more designs. This method is a special case of correlated systematic uncertainties. Consider a burst test for an improved container design. The improvement in the design can be expressed as the fraction

$$R = \frac{P_n}{P_b}$$

where P_n is the burst pressure of the new design and P_b is the burst pressure of the original or base design.

Table 8.1 provides burst tests for two different programs. In the first test program, different pressure transducers were used in the tests on the two designs. There were no correlated systematic uncertainties common between these two transducers. In the second program, the same pressure transducer was used for both tests; therefore, the systematic uncertainty was the same and was correlated for the two test measurements.

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TABLE 8.1
BURST PRESSURES (EXAMPLE 8.1)

	P_b Base Design 10^6 Pa	P_n Improved Design 10^6 Pa	B_P 10^6 Pa
Program 1			
Meter #1	40.0	. . .	0.2
Meter #2	. . .	52.0	0.2
Program 2			
Meter #3	42.0	54.7	0.5

Program 1 (no correlated systematic uncertainties):

$$R = \frac{52.0}{40.0} = 1.30$$

$$\theta_b = \frac{-R}{P_b} = -0.0325\ (10^6\ \text{Pa})^{-1}$$

$$\theta_n = \frac{R}{P_n} = 0.0250\ (10^6\ \text{Pa})^{-1}$$

$$B_R^2 = [(-0.0325)(10^6\ \text{Pa})^{-1}\ (0.2)(10^6\ \text{Pa})]^2 + [(0.025)(10^6\ \text{Pa})^{-1}\ (0.2)(10^6\ \text{Pa})]^2$$

$$B_R = 0.0082$$

Program 2 (correlated systematic uncertainties):

$$R = \frac{54.7}{42.0} = 1.30$$

$$\theta_b = -0.0310\ (10^6\ \text{Pa})^{-1}$$

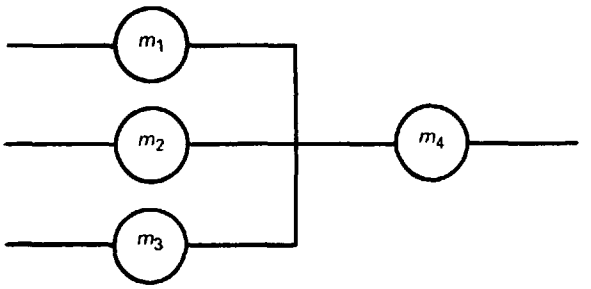
$$\theta_n = 0.0238\ (10^6\ \text{Pa})^{-1}$$

$$B_R^2 = [(-0.0310)(10^6\ \text{Pa})^{-1}\ (0.5)(10^6\ \text{Pa})]^2 + [(0.0238)(10^6\ \text{Pa})^{-1}\ (0.5)(10^6\ \text{Pa})]^2 + 2(-0.0310)(10^6\ \text{Pa})^{-1}\ (0.0238)(10^6\ \text{Pa})^{-1}\ (0.5)(10^6\ \text{Pa})(0.5)(10^6\ \text{Pa})$$

$$B_R = 0.0036$$

This example demonstrates the strength of the back-to-back testing technique using the same instrumentation. Even though the pressure transducer in program 2 had a systematic uncertainty of more than twice those of the transducers in program 1, the systematic uncertainty of the result for program 2 was less than half of that for program 1.

Example 8.2 (adapted from [8]): Consider the piping arrangement shown below with the four flowmeters:



From conservation of mass, a balance check would yield

$$z = m_4 - m_1 - m_2 - m_3 = 0$$

If the errors in the flow rate measurements are predominantly systematic, then for the balance check to be satisfied

$$|z| \le B_z$$

Consider the case where the dominant systematic errors are from the calibration standard and from the calibration curve fit. The calibration standard systematic uncertainty for each flowmeter is a percent of reading and is ±3 lbm/hr for the three small meters and ±9 lbm/hr for the large meter. The curve fit systematic uncertainty for each meter is ±1 lbm/hr. Note that for this example the derivatives for Eq. (8.2) are

$$\theta_{m_1} = \theta_{m_2} = \theta_{m_3} = -1$$

and

$$\theta_{m_4} = 1$$

Case 1 — Each flowmeter calibrated against a different standard.

In this case, all of the systematic uncertainties are uncorrelated with the systematic uncertainty for the three small flowmeters determined as

$$B_{m_i}\ (i = 1,2,3) = \left[(3)^2 + (1)^2\right]^{1/2} = 3.16\ \text{lbm/hr}$$

and the systematic uncertainty for the large flowmeter calculated as

$$B_{m_4} = \left[(9)^2 + (1)^2\right]^{1/2} = 9.06\ \text{lbm/hr}$$

Using Eq. (7.12), the systematic uncertainty for z is

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$$B_z = [(\theta_1 B_{m_1})^2 + (\theta_2 B_{m_2})^2 + (\theta_3 B_{m_3})^2 + (\theta_4 B_{m_4})^2]^{1/2}$$

or

$$B_z = [(B_{m_1})^2 + (B_{m_2})^2 + (B_{m_3})^2 + (B_{m_4})^2]^{1/2}$$
$$= 10.6 \text{ lbm/hr}$$

Case 2 — Flowmeters 1, 2, and 3 calibrated against the same standard and flowmeter 4 calibrated against a different standard.

In this case, the systematic uncertainties are

$$B_{m_1} = B_{m_2} = B_{m_3} = 3.16 \text{ lbm/hr}$$

$$B_{m_4} = 9.06 \text{ lbm/hr}$$

and

$$B_{m_1 m_2} = B_{m_1 m_3} = B_{m_2 m_3} = (3 \text{ lbm/hr})(3 \text{ lbm/hr})$$

Using Eq. (8.2) for four measurements with three of them having correlated systematic uncertainties, the systematic uncertainty for z becomes

$$B_z = [(\theta_1 B_{m_1})^2 + (\theta_2 B_{m_2})^2 + (\theta_3 B_{m_3})^2 + (\theta_4 B_{m_4})^2 + 2\theta_1\theta_2 B_{m_1 m_2} + 2\theta_1\theta_3 B_{m_1 m_3} + 2\theta_2\theta_3 B_{m_2 m_3}]^{1/2}$$

or

$$B_z = [(B_{m_1})^2 + (B_{m_2})^2 + (B_{m_3})^2 + (B_{m_4})^2 + 2B_{m_1 m_2} + 2B_{m_1 m_3} + 2B_{m_2 m_3}]^{1/2}$$
$$B_z = 12.9 \text{ lbm/hr}$$

Note that in this case the signs for all of the correlated terms are positive because all of the derivatives of z, with respect to m_1, m_2, and m_3, are negative. If flowmeters 1, 2, and 3 are calibrated against the same standard, and flowmeter 4 is calibrated against a different standard, the systematic uncertainty for z is larger than if all the meters had been calibrated against different standards (Case 1).

Case 3 — Flowmeters 1, 2, 3, and 4 calibrated against the same standard.

In this case, the systematic uncertainties are

$$B_{m_1} = B_{m_2} = B_{m_3} = 3.16 \text{ lbm/hr}$$

and

$$B_{m_4} = 9.06 \text{ lbm/hr}$$

with

$$B_{m_1 m_2} = B_{m_1 m_3} = B_{m_2 m_3} = (3 \text{ lbm/hr})(3 \text{ lbm/hr})$$

and

$$B_{m_1 m_4} = B_{m_2 m_4} = B_{m_3 m_4} = (3 \text{ lbm/hr})\ (9 \text{ lbm/hr})$$

Using Eq. (8.2) for four measurements, all with correlated systematic uncertainties, the systematic uncertainty for z is

$$B_z = [(\theta_1 B_{m_1})^2 + (\theta_2 B_{m_2})^2 + (\theta_3 B_{m_3})^2 + (\theta_4 B_{m_4})^2 + 2\theta_1\theta_2 B_{m_1 m_2} + 2\theta_1\theta_3 B_{m_1 m_3} + 2\theta_1\theta_4 B_{m_1 m_4} + 2\theta_2\theta_3 B_{m_2 m_3} + 2\theta_2\theta_4 B_{m_2 m_4} + 2\theta_3\theta_4 B_{m_3 m_4}]^{1/2}$$

or

$$B_z = [(B_{m_1})^2 + (B_{m_2})^2 + (B_{m_3})^2 + (B_{m_4})^2 + 2B_{m_1 m_2} + 2B_{m_1 m_3} - 2B_{m_1 m_4} + 2B_{m_2 m_3} - 2B_{m_2 m_4} - 2B_{m_3 m_4}]^{1/2}$$
$$B_z = 2.0 \text{ lbm/hr}$$

Note the signs for each of the correlated terms.

For this example, calibrating all the flowmeters against the same standard will yield the minimum systematic uncertainty for z. In general, correlated systematic uncertainties can either decrease, increase, or have no effect on the systematic uncertainty of the result, depending on the form of the data reduction equation and on which parameters have correlated systematic errors.

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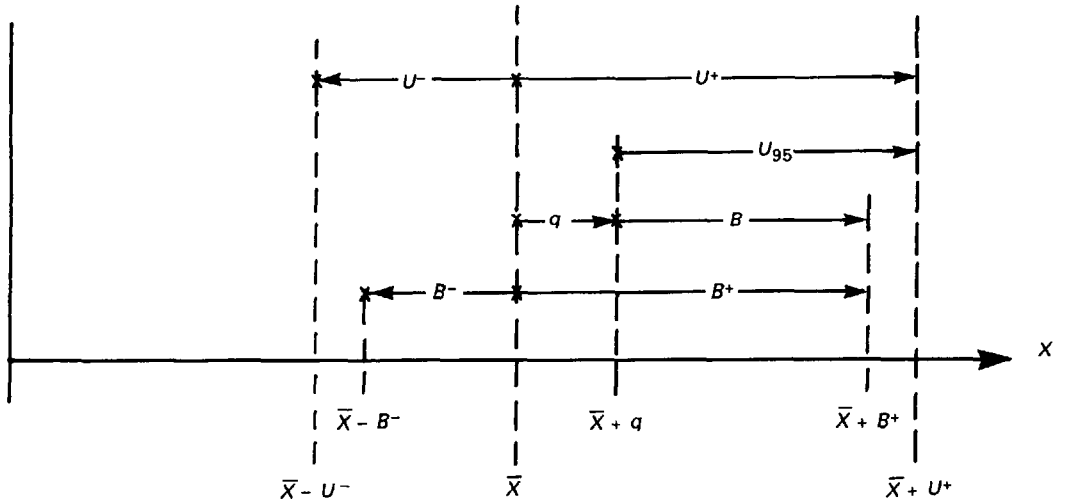

FIG. 8.1 SCHEMATIC RELATION BETWEEN PARAMETERS CHARACTERIZING NONSYMMETRIC UNCERTAINTY

8.2 NONSYMMETRIC SYSTEMATIC UNCERTAINTY

In some experiments, physical models can be used to essentially replace the asymmetric uncertainties with symmetric uncertainties in additional experimental variables. If this can be done, then it should be, but if not, then the method of para. 8.2.1 should be used. This paragraph presents a method for determining nonsymmetric uncertainty intervals in these cases [10].

8.2.1 Nonsymmetric Systematic Uncertainty Intervals in a Measured Variable. If the distribution of the systematic uncertainty associated with a measured variable is not centered on the measured value, then the overall uncertainty interval for the measured variable will not be centered on the measured value of the variable. In this case, the following procedure should be employed for constructing a nonsymmetric uncertainty interval for the measured variable (see Fig. 8.1):

(a) Specify an interval $(\overline{X} - B^-, \overline{X} + B^+)$ relative to the measured value within which one may expect the systematic uncertainty to fall with 95% confidence. The interval that is specified should contain the measured value of the variable.

(b) Define q as the difference between the center of the interval specified in (*a*) and the measured value.

(c) Estimate the component of systematic uncertainty B by one-half of the length of the interval specified in (*a*).

(d) Estimate the total uncertainty for the measurement using the standard formula:

$$U_{95} = 2[(B/2)^2 + (S_{\overline{X}})^2]^{1/2} \qquad (8.4)$$

(e) Estimate a 95% confidence interval for the measured variable using:

$$[\overline{X} + q] \pm U_{95} \qquad (8.5)$$

(f) Express the final result as an asymmetric 95% confidence interval for the measured variable as follows, where the lower limit on this interval is given by:

$$\overline{X}_{\text{lower limit}} = \overline{X} - (U_{95} - q) = \overline{X} - U^- \qquad (8.6)$$

and the upper limit on this interval is given by:

$$\overline{X}_{\text{upper limit}} = \overline{X} + (U_{95} + q) = \overline{X} + U^+ \qquad (8.7)$$

Example: Suppose a thermocouple is being used to measure the temperature of a gas stream, but the user of the thermocouple believes there may be a tendency for the thermocouple to provide a temperature reading that is lower than the actual gas tempera-

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ture due to a radiative heat transfer mechanism. The user does not have enough information to properly correct the thermocouple reading for these effects, but wishes to account for them in an uncertainty analysis. From a sample of more than 30 readings using the thermocouple, the user finds that $\overline{X} = 534.7°C$ and $S_{\overline{X}} = 2.4°C$. If the user believes that the true gas temperature may be as much as 10°C higher than $\overline{X}$ due to radiation effects, then how should the user specify a nonsymmetric confidence interval accounting for this nonsymmetric systematic uncertainty?

Step 1: Specify an interval (corresponding to 95% confidence) for the systematic uncertainty in question. In this case, the user of the thermocouple believes that the true gas temperature falls between the average measured with the thermocouple, $\overline{X} = 534.7°C$, and a value that is 10°C higher than $\overline{X}$, i.e., 544.7°C.

Step 2: Determine q, the difference between the center of the interval specified in Step 1 and the value measured with the thermocouple. In this case, $q = (544.7°C + 534.7°C)/2 - 534.7°C = 5°C$.

Step 3: Estimate B by one half of the length of the interval specified in Step 1. In this case, $B = (10°C)/2 = 5°C$.

Step 4: Estimate the total uncertainty for the measurement, U_{95}. In this case, $U_{95} = 2[(\{5°C\}/2)^2 + (2.4°C)^2]^{1/2} = 6.9°C$.

Step 5: Estimate a 95% confidence interval for the average using $[\overline{X} + q] \pm U_{95}$. In this case, this 95% confidence interval is given by $[534.7°C + 5°C] \pm 6.9°C$.

Step 6: Express the final result as an asymmetric 95% confidence interval for the average using $\overline{X} \pm (U_{95} \pm q)$. In this case, this 95% confidence interval is given by $534.7°C \pm (6.9°C \pm 5°C)$.

Note that in this example the asymmetric 95% confidence interval for the average extends from $\overline{X}_{\text{lower limit}} = 534.7°C - (6.9°C - 5°C) = 532.8°C$, i.e., 1.9°C below the average $\overline{X}$, to $\overline{X}_{\text{upper limit}} = 534.7°C + (6.9°C + 5°C) = 546.6°C$, i.e., 11.9°C above the average $\overline{X}$. This result corresponds to the user's intuition that the true value of the gas temperature may be higher than the value measured with the thermocouple.

8.2.2 Nonsymmetric Systematic Uncertainty Interval for a Derived Result. A nonsymmetric systematic uncertainty in a measured variable may also result in a nonsymmetric uncertainty interval in a derived result. The following procedure should be employed for propagating the nonsymmetric uncertainties in a set of measured variables to a derived result (see Fig. 8.2):

(a) Determine $\overline{X}_i$, $U_{95,\overline{X}i}$, and q_i for each average $\overline{X}_i$ that contributes to the determination of the derived result, $r(\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_n)$.

(b) Define q_r as the difference between $r(\overline{X}_1 + q_1, \overline{X}_2 + q_2, \ldots, \overline{X}_n + q_n)$ and $r(\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_n)$.

(c) Determine the sensitivity coefficient, θ_i, for each average $\overline{X}_i$ that contributes to the derived result by the standard means. If a sensitivity coefficient depends on the values of any averages, i.e., $\theta_i = \theta_i(\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_n)$, then it should be evaluated at the point $(\overline{X}_1 + q_1, \overline{X}_2 + q_2, \ldots, \overline{X}_n + q_n)$.

(d) Estimate the total uncertainty for the derived result, $U_{95,r}$, using the standard formula:

$$U_{95,r} = [(\theta_1 U_{95,\overline{X}1})^2 + (\theta_2 U_{95,\overline{X}2})^2 + \ldots + (\theta_n U_{95,\overline{X}n})^2]^{1/2} \quad (8.8)$$

(e) Estimate a 95% confidence interval for the derived result using:

$$r(\overline{X}_1 + q_1, \overline{X}_2 + q_2, \ldots, \overline{X}_n + q_n) \pm U_{95,r} \quad (8.9)$$

(f) Express the final result as an asymmetric 95% confidence interval for the derived result as follows:

$$r(\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_n) \pm (U_{95,r} \pm q_r) \quad (8.10)$$

where the lower limit on this interval is given by

$$r_{\text{lower limit}} = r(\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_n) - (U_{95,r} - q_r) = r - U_r^- \quad (8.11)$$

and the upper limit on this interval is given by

$$r_{\text{upper limit}} = r(\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_n) + (U_{95,r} + q_r) = r + U_r^+ \quad (8.12)$$

Example: Suppose the user of the thermocouple in the example in para. 8.2.1 wishes to use this gas temperature to estimate the speed of sound for the gas using the following relation:

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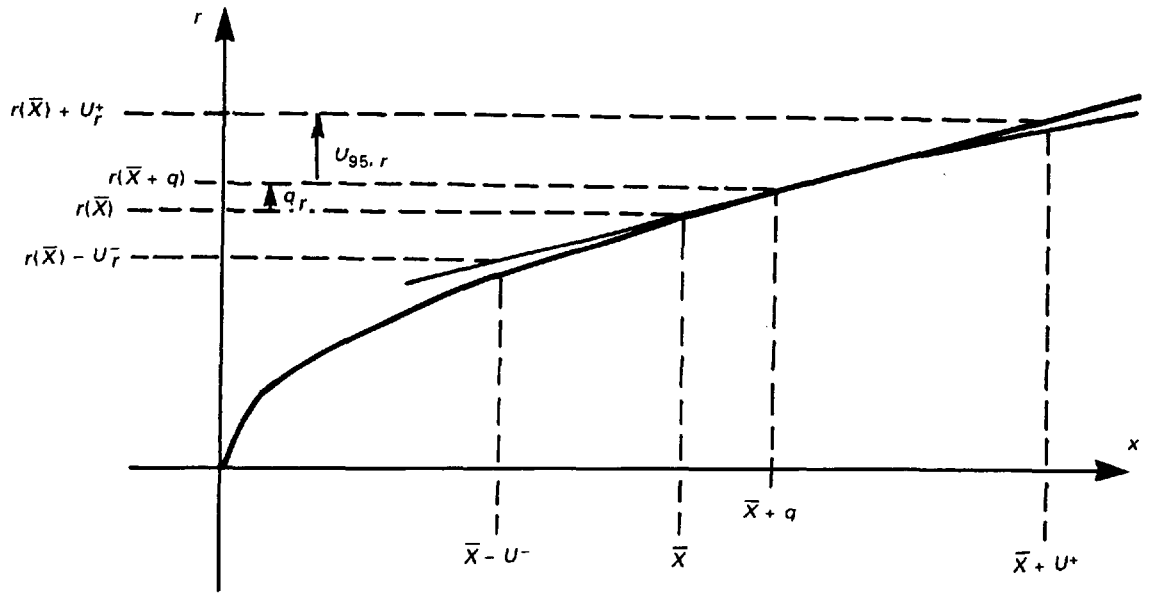

FIG. 8.2 RELATION BETWEEN PARAMETERS CHARACTERIZING NONSYMMETRIC UNCERTAINTY

$$c = [kRT]^{1/2}$$

where k, the ratio of specific heats, and R, the gas constant for the gas, are taken to be constant with negligible uncertainty and T is the measured value of the absolute temperature. In this case, $T = (\overline{X} + 273.2)$ K, where $\overline{X}$ is the average value of the temperature of the gas using the thermocouple. What is the uncertainty interval for c?

Step 1: Determine T, $U_{95,T}$, and q_T for the measured variable T. In this case, $T = 807.7$ K, $U_{95,T} = 6.9$ K, and $q_T = 5$ K.

Step 2: Determine q_c, the difference between $c(T + q_T)$ and $c(T)$. In this case, $q_c = [kR]^{1/2}$ $[(812.7\ K)^{1/2} - (807.7\ K)^{1/2}] = [kR]^{1/2}$ $(0.0878\ K^{1/2})$.

Step 3: Determine the sensitivity coefficient, θ_T, for the measured variable T. In this case, $\theta_T = (1/2)\ [kR/T]^{1/2}$. Since this sensitivity coefficient depends on T, it should be evaluated at $T + q_T = 812.7$ K, so that here $\theta_T = (1/2)[kR/(812.7\ K)]^{1/2} = [kR]^{1/2}$ $(0.0175\ K^{-1/2})$.

Step 4: Estimate the total uncertainty for the derived result, $U_{95,c}$. In this case, $U_{95,c} = [\{[kR]^{1/2}\ (0.0175\ K^{-1/2})\ (6.9\ K)\}^2]^{1/2} = [kR]^{1/2}$ $(0.1208\ K^{1/2})$.

Step 5: Estimate a 95% confidence interval for the derived result using $c(T + q_T) \pm U_{95,c}$. In this case, this 95% confidence interval is given by $[kR]^{1/2}$ $[812.7\ K]^{1/2} \pm [kR]^{1/2}$ $(0.1208\ K^{1/2})$.

Step 6: Express the final result as an asymmetric 95% confidence interval using $c(T) \pm (U_{95,c} \pm q_c)$. In this case, this 95% confidence interval is given by $[kR]^{1/2}$ $[807.7\ K]^{1/2} \pm \{[kR]^{1/2}\ (0.1208\ K^{1/2}) \pm [kR]^{1/2}\ (0.0878\ K^{1/2})\}$.

In this example, the uncertainty interval for the speed of sound of the gas extends from 0.12% below to 0.73% above the value for the speed of sound assessed using the measured value of the temperature.

8.3 FOSSILIZATION OF CALIBRATIONS

Definition of the Measurement Process is a prerequisite for determining measurement uncertainty estimates. For example, different Defined Measurement Processes for the same test will result in different estimates of measurement uncertainty. Elemental errors are classified as random if they add scatter to a result. If they do not, they are systematic. Final

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classification is dependent on the Defined Measurement Process.

Calibration errors are frequently reclassified as a result of the Defined Measurement Process. As an example, for test measurements involving multiple calibrations (i.e., calibrations performed prior to each of the multiple tests or periodically throughout a test program), the calibration process random error will introduce scatter in the test data set and therefore should remain classified as a random error source or random uncertainty in the uncertainty analysis. However, for test measurements involving a single calibration (i.e., calibrations performed only once during a test program, and all data samples processed using the same calibration constants), the calibration process random error does not have an opportunity to introduce scatter in the test data set and therefore should be reclassified (fossilized) as systematic error during the final uncertainty analysis. In this case, the calibration random uncertainty should be treated as another systematic elemental uncertainty and combined with the other calibration systematic uncertainty to obtain the total systematic uncertainty for the calibration process as:

$$B_c^* = 2\sqrt{(b_1/2)^2 + (b_2/2)^2 + \ldots (S_{\bar{c}})^2} \qquad (8.13)$$

where

B_c^* = total systematic uncertainty of the single calibration process

$b_1, b_2 \ldots$ = elemental systematic uncertainty components of the calibration process

S_c = standard deviation of the calibration process, an estimated standard deviation of the random error components of the calibration process

$$S_{\bar{c}} = \frac{S_c}{\sqrt{n}}$$

n = number of calibration points within a single calibration

A more general form of Eq. (8.13) for a single calibration is

$$B_c^* = 2[(B_c/2)^2 + (S_{\bar{c}})^2]^{1/2} \qquad (8.14)$$

where

B_c = calibration process initial systematic uncertainty before reclassifying (fossilizing) the calibration random uncertainty into systematic uncertainty

To illustrate the fossilization process, consider a master flowmeter installed in line with a test flowmeter for use in establishing a calibration correction for the test meter. As a result of the calibration process, the systematic uncertainty of the test meter is replaced by that of the master meter. The calibration process random uncertainty is a function of the initial random uncertainties in both the master and test meters. When a data set of interest from the test meter involves multiple calibrations, the calibration process random uncertainty will cause scatter in the individual test meter data samples and thus should remain classified as calibration random uncertainty in the uncertainty analysis. However, when a data set from the test meter involves only a single calibration, the calibration process random uncertainty is common to all data samples and thus manifests itself in the data set as a systematic uncertainty (becomes fossilized). In this case, the initial random and systematic uncertainties of the calibration process should be combined and carried forward as a systematic uncertainty.

Fossilization of calibration random uncertainty can occur at any or all levels of calibration, from NIST (National Institute of Standards and Technology) to the test application, depending on the defined calibration, or measurement processes, or both.

Sometimes, multiple calibrations are performed but the results are averaged into a single set of calibration constants for use in processing all data samples (e.g., pre- and postcalibrations). In this case, a portion of the random uncertainty still becomes fossilized into systematic uncertainty. The magnitude has been reduced by having averaged multiple calibrations. The term $S_{\bar{c}}$ in Eq. (8.14) should be reduced by dividing by $\sqrt{N}$. Thus a more general form of Eq. (8.14) would be:

$$B_c^* = 2[(B_c/2)^2 + (S_{\bar{c}}/\sqrt{N})^2]^{1/2} \qquad (8.15)$$

where

N = number of repeated independent (single) calibrations averaged and used in obtaining a single set of calibration constants common to all samples within the test data set.

8.4 SPATIAL VARIATION [11]

Measurement requirements for a performance test are often such that an average measurement of individual parameters is required. Most instrumentation, however, yields a point measurement of a

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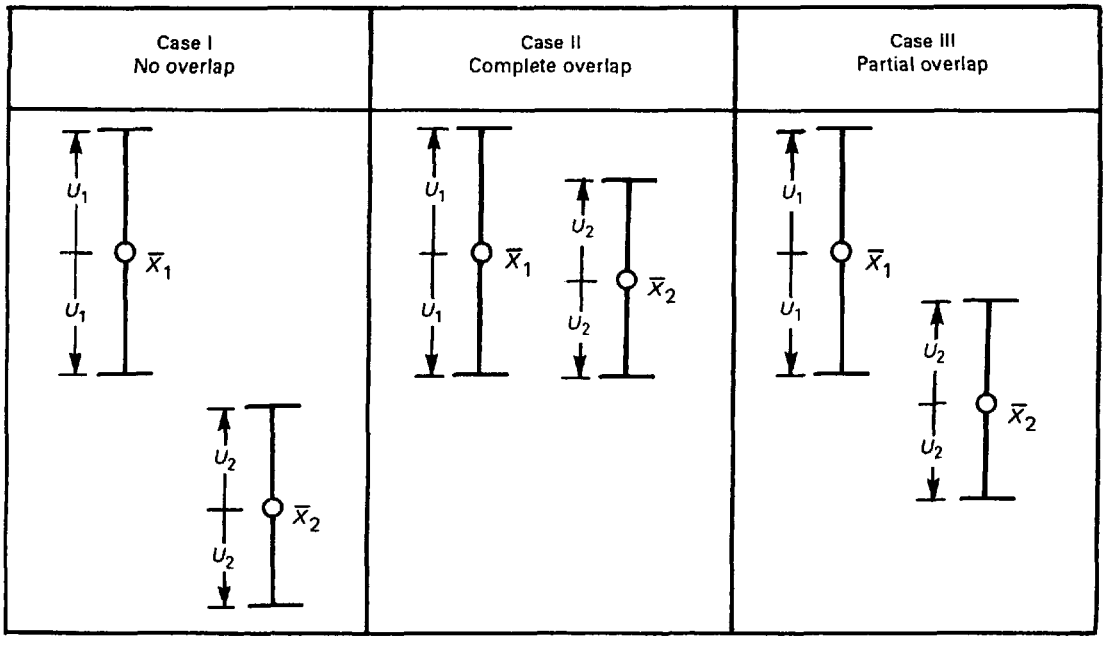

FIG. 8.3 THREE POSTTEST CASES

parameter rather than an average measurement. While this point characteristic may be useful for other purposes, it raises a problem in determining performance level. In many instances, the quantity measured varies in space, making the point measurement inadequate. Thus, it often is necessary to install several measurement sensors at different spatial locations to account for spatial variations of the parameter being measured. Spatial variation effects are considered errors of method (see para. 5.4.4).

A simple illustration of the impact of spatial variation is the measurement of average velocity of an incompressible fluid in a pipe. At low Reynolds numbers, the flow is laminar and has a parabolic velocity profile. One measurement will not give the true average value directly. For example, the velocity at the center of the pipe is twice the average velocity. This is in contrast to the situation at high Reynolds numbers, where the flow is turbulent. At higher Reynolds numbers, the profile approaches uniformity and any measurement will yield a good estimate of the average velocity. Usually, test conditions vary between these two extremes and it is not possible to correct the readings in a simple manner. Circumferential variations may also be present. Therefore, other approaches are chosen, such as installing multiple sensors and averaging the outputs. An example of this is an averaging pressure probe often used to measure average velocity at a cross section of pipe.

The uncertainty of this mean velocity would be calculated by considering it to be a determined result. An additional systematic uncertainty may need to be assigned to the mean result to account for the possible difference between the determined mean and the true mean.

8.5 POSTTEST ANALYSIS OF REDUNDANT MEANS

Posttest analysis is required to establish the actual test uncertainty interval. It is also used to confirm the pretest uncertainty estimates, or to identify problems, or both. When redundant instrumentation or calculation methods are available, the individual results and their uncertainty intervals should be compared with each other and with the pretest uncertainty analysis.

When comparing redundant means ($\bar{X}_1$ and $\bar{X}_2$) and their uncertainty intervals, the three cases illustrated in Fig. 8.3 need to be considered.

Case 1. A problem clearly exists when there is no overlap between uncertainty intervals. Either uncertainty intervals have been grossly underesti-

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mated, an error exists in the measurements, or the true value is not constant. Investigation to identify bad readings, overlooked or underestimated systematic uncertainty, etc., is necessary to resolve this discrepancy.

Case 2. When the uncertainty intervals completely overlap, as in Case 2, one can be confident that there has been a proper accounting of all major uncertainty components. The smaller uncertainty interval $\overline{X}_2 \pm U_2$ is wholly contained in the larger interval $\overline{X}_1 \pm U_1$. Since the individual measurements are valid, the weighing method technique described in para. A.2 can be used to obtain a better estimate of the true value than either of the individual measurements.

Case 3. Case 3, where a partial overlap of the uncertainty intervals exists, is the most difficult to analyze. For both measurements and both uncertainty intervals to be correct, the true value must lie in the region where the uncertainty intervals overlap. Consequently, the larger the overlap, the more confidence we have in the validity of the measurements and the estimate of the uncertainty intervals. As the difference between the two measurements increases, the overlap region shrinks. Standard statistical hypothesis testing may be used to evaluate the significance of differences observed.

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SECTION 9 — STEP-BY-STEP CALCULATION PROCEDURE

9.1 GENERAL CONSIDERATIONS

It is recommended that an uncertainty analysis, following the methods of Sections 4 through 8 of this Supplement, be conducted before and after each test, according to the procedure given below.

The pretest analysis (see para. 4.4) is used to determine if the test result can be measured with sufficient accuracy, i.e., the predicted uncertainty should be smaller than the required uncertainty. It may also be used to compare alternative instrumentation systems and test designs and to determine corrective action if the predicted uncertainty is unacceptably large. Furthermore, it may be used to evaluate the need for calibration. The posttest analysis (see para. 4.5) validates the pretest analysis, provides data for validity checks, and provides a statistical basis for comparing test results.

9.2 CALCULATION PROCEDURE

(a) Define Measurement Process (see Section 5)

(1) Review test objectives and test duration.

(2) List all independent measurement parameters and their nominal levels.

(3) List all calibrations and instrument setups that will affect each parameter. Be sure to check for uncertainties in measurement system components that affect two or more measurements simultaneously (correlated uncertainties).

(4) Define the functional relationship between the independent measurement parameters and the test result.

(b) List Elemental Error Sources (see para. 5.4)

(1) Make a complete and exhaustive list of all possible test uncertainty sources for all parameters.

(c) Calculate the Systematic and Random Standard Deviation for Each Parameter (see paras. 6.1 and 6.2)

(d) Propagate the Systematic and Random Standard Deviations (see paras. 7.1 through 7.4)

(1) The systematic and random (sample) standard deviations of the independent parameters are propagated separately all the way to the final result.

(2) Propagation of the standard deviations is done, according to the functional relationship defined in step (a)(4) above, by using the Taylor series method (see Appendix C). This requires a calculation of the sensitivity factors, either by differentiation or by computer perturbation.

(e) Calculate Uncertainty (see para. 7.5)

(1) Combine the systematic and random uncertainties to obtain the total uncertainty.

(f) Report

The uncertainty analysis for each calculated result should be reported on two tables. The first is a detailed report which displays all the information used in the calculation of the nominal value and uncertainty of the result. The second is a table that summarizes the uncertainty information at the result level.

For most uncertainty analyses, all measured parameters will have symmetric systematic uncertainties and large degrees of freedom. For some analyses, one or more of the systematic uncertainties may be nonsymmetric (see para. 8.2), and for other analyses, the degrees of freedom may be small for some of the uncertainties (see Appendix B).

The detailed report table should include, as a minimum, the following information for each parameter used in the calculation of the result:

- Symbol used in the calculations
- Description
- Units
- Nominal value (average of measurements), $\overline{X}_i$
- Systematic uncertainty, B_i
- Random (sample) standard deviation, $S_{\overline{x},i}$
- Sensitivity, θ_i
- Systematic contribution to the total uncertainty of the result $\left(\frac{B_i}{2}\theta_i\right)^2$
- Random contribution to the total uncertainty of the result $(S_{\overline{x},i}\,\theta_i)^2$

The summary report table should display the following information associated with the result:

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TABLE 9.1
TABLE OF DATA (EXAMPLE 10.2)

Independent Parameters								
Symbol	Description	Units	Nominal Value	Absolute Systematic Uncertainty (B_i)	Absolute Standard Deviation of the Mean ($S_{\bar{x},i}$)	Absolute Sensitivity (θ_i)	Absolute Systematic Uncertainty Contribution $\left(\frac{B_i}{2}\theta_i\right)^2$	Absolute Random Uncertainty Contribution $(S_{\bar{x},i}\,\theta_i)^2$
C	Discharge coefficient	...	0.964	0.0075	0	140	2.78E-1	0
d	Throat diameter	Inches	4	0.001	0	86.2	1.86E-3	0
D	Inlet diameter	Inches	6	0.002	0	−11.4	1.29E-4	0
ρ	Water density at 60°F [25]	lbm/ft^3	62.37	0.004	0.002	1.11	4.92E-6	4.92E-6
h	Differential pressure head across venturi (at 68°F)	in. H_2O	100	0.3	0.4	0.6919	1.06E-2	7.66E-2

TABLE 9.2
SUMMARY OF DATA (EXAMPLE 10.2)

Calculated Result						
Symbol	Description	Units	Calculated Value	Absolute Systematic Uncertainty, B_R $\left(2\sqrt{\sum\left(\frac{B_R}{2}\theta_i\right)^2}\right)$	Absolute Random Uncertainty, $2S_R$ $\left(2\sqrt{\sum(S_{\bar{x},i}\,\theta_i)^2}\right)$	Absolute Uncertainty, U_R $\left(\sqrt{B_R^2 + (2S_R)^2}\right)$
$\dot{m}$	Mass flow rate	lbm/s	138.386	1.06	0.55	1.21

- Symbol used in the calculations
- Description
- Units
- Calculated value (calculated using nominal values for measured parameters), R
- Systematic uncertainty, B_R, $\left(2\sqrt{\sum_{i=1}^{N}\left(\frac{B_i}{2}\theta_i\right)^2}\right)$
- Random uncertainty, $2S_R$, $\left(2\sqrt{\sum_{i=1}^{N}(S_{\bar{x},i}\,\theta_i)^2}\right)$
- Total uncertainty, U_R, $\left(2\sqrt{\left(\frac{B_R}{2}\right)^2 + S_R^2}\right)$

Tables 9.1 and 9.2 are examples of the detailed and summary tables, respectively, that should be used for the reporting of uncertainty analyses for which all degrees of freedom are large and all systematic uncertainties are symmetric and uncorrelated. These tables are taken from Example 10.2. The calculations can be made and the values displayed on either a relative or an absolute basis — nothing in the procedure changes — but the headings in the report tables should always specify which basis is being used.

When one or more of the parameters has a systematic uncertainty that is nonsymmetric, the report tables will have to be expanded. The detailed report table for an uncertainty analysis that includes systematic uncertainties that are nonsymmetric will be set up with the following columns:

- Symbol used in the calculations
- Description
- Units
- Nominal value (average of measurements), $\bar{X}_i$
- Systematic uncertainty, B_i

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- Offset from the center of the systematic uncertainty interval, q_i (which is 0 for symmetric uncertainties)
- Random (sample) standard deviation, $S_{\bar{x},i}$
- Sensitivity, θ_i
- Systematic contribution to the total uncertainty of the result, $\left(\frac{B_i}{2}\theta_i\right)^2$
- Random contribution to the total uncertainty of the result, $(S_{\bar{x},i}\,\theta_i)^2$

The summary report table for a nonsymmetric uncertainty analysis will contain the following columns:

- Symbol used in the calculations
- Description
- Units
- Calculated value (calculated using nominal values for measured parameters), R
- Systematic uncertainty, B_R, $\left(2\sqrt{\sum_{i=1}^{N}\left(\frac{B_i}{2}\theta_i\right)^2}\right)$
- Offset from the center of the systematic uncertainty interval, q_R
- Random uncertainty, $2S_R$, $\left(2\sqrt{\sum_{i=1}^{N}(S_{\bar{x},i}\,\theta_i)^2}\right)$
- Total upper uncertainty limit, U_R^+, $\left(2\sqrt{\left(\frac{B_R}{2}\right)^2+S_R^2}+q_R\right)$
- Total lower uncertainty limit, U_R^-, $\left(2\sqrt{\left(\frac{B_R}{2}\right)^2+S_R^2}-q_R\right)$

For symmetric uncertainty analyses in which one or more of the uncertainties has small degrees of freedom (i.e., less than 10), the detailed report table will contain the following information for each of the measured parameters:

- Symbol used in the calculations
- Description
- Units
- Nominal value (average of measurements), $\bar{X}_i$
- Systematic uncertainty, B_i
- Random (sample) standard deviation, $S_{\bar{x},i}$
- Number of measurements used in calculation of nominal value, N_i
- Degrees of freedom associated with systematic uncertainty, ν_{B_i}
- Degrees of freedom associated with the random uncertainty, $\nu_{S_{\bar{x},i}}$
- Sensitivity, θ_i
- Systematic contribution to the total uncertainty of the result, $\left(\frac{B_i}{2}\theta_i\right)^2$
- Random contribution to the total uncertainty of the result, $(S_{\bar{x},i}\,\theta_i)^2$
- Value of term associated with the systematic uncertainty in the denominator of the Welch-Satterthwaite equation used to calculate composite degrees of freedom for the result, $\left(\frac{\left(\theta_i\left(\frac{B_i}{2}\right)\right)^4}{\nu_{B_i}}\right)$
- Value of term associated with the random uncertainty in the denominator of the Welch-Satterthwaite equation used to calculate composite degrees of freedom for the result, $\left(\frac{(\theta_i\,S_{\bar{x},i})^4}{\nu_{S_{\bar{x},i}}}\right)$

The summary report tables for an uncertainty analysis that includes parameters with small degrees of freedom would have the following columns:

- Symbol used in the calculations
- Description
- Units
- Calculated value (calculated using nominal values for measured parameters), R
- Systematic uncertainty, $\frac{tB_R}{2}$
- Random uncertainty, tS_R
- Composite degrees of freedom, ν_R (see Appendix B for formula)
- Student's t associated with the composite degrees of freedom, t
- Total uncertainty, U_R, $\left(t\sqrt{\left(\frac{B_R}{2}\right)^2+S_R^2}\right)$

Tables 9.3 and 9.4 show the detailed report for Example 10.2.

Table 9.5 shows the results table that would be used for this case. As Example 10.2 involves no parameters having nonsymmetric uncertainties or small degrees of freedom, N/A has been entered into the columns not required for this analysis.

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TABLE 9.3
REPORT OF RESULTS (EXAMPLE 10.2)

Independent Parameters									
Symbol	Description	Units	Nominal Value	Number of Measurements Used to Calculate Nominal Value (N_i)	Absolute Systematic Uncertainty (B_i)	Degrees of Freedom for Systematic Uncertainty (ν_{Bi})	Absolute Offset of Uncertainty Interval (q_i)	Absolute Standard Deviation of the Mean ($S_{\bar{x},i}$)	Degrees of Freedom for Standard Deviation of the Mean ($\nu_{S_{\bar{x}},i}$)
C	Discharge coefficient	. . .	0.984	N/A	0.0075	N/A	0	0	N/A
d	Throat diameter	Inches	4	N/A	0.001	N/A	0	0	N/A
D	Inlet diameter	Inches	6	N/A	0.002	N/A	0	0	N/A
ρ	Water density at 60°F [25]	lbm/ft^3	62.37	N/A	0.004	N/A	0	0.002	N/A
h	Differential pressure head across venturi (at 68°F)	in. H_2O	100	N/A	0.3	N/A	0	0.4	N/A

TABLE 9.4
REPORT OF RESULTS (EXAMPLE 10.2)

Independent Parameters								
Symbol	Description	Units	Nominal Value	Absolute Sensitivity (θ_i)	Absolute Systematic Uncertainty Contribution $\left(\frac{B_i}{2}\theta_i\right)^2$	Absolute Random Uncertainty Contribution $(S_{\bar{x},i}\,\theta_i)^2$	Denominator Term for Systematic Uncertainty for Welch-Satterthwaite Formula $\left(\frac{\left(\theta_i \frac{B_i}{2}\right)^4}{\nu_{Bi}}\right)$	Denominator Term for Random Uncertainty for Welch-Satterthwaite Formula $\left(\frac{(\theta_i S_{\bar{x},i})^4}{\nu_{S\bar{x},i}}\right)$
C	Discharge coefficient	. . .	0.984	1.40	2.78E-1	0	N/A	N/A
d	Throat diameter	Inches	4	86.2	1.86E-3	0	N/A	N/A
D	Inlet diameter	Inches	6	−11.3	1.29E-4	0	N/A	N/A
ρ	Water density, at 60°F [25]	lbm/ft^3	62.37	1.11	4.92E-6	4.92E-6	N/A	N/A
h	Differential pressure head across venturi (at 68°F)	in. H_2O	100	0.692	1.06E-2	7.66E-2	N/A	N/A

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TABLE 9.5
SUMMARY OF RESULTS (EXAMPLE 10.2)

			Calculated Result									
Symbol	Description	Units	Calculated Value	Absolute Systematic Uncertainty, B_R $\left(t\sqrt{\sum\left(\frac{B_i}{2}\theta_i\right)^2}\right)$	Offset From Center of Systematic Uncertainty Interval, q_R	Absolute Random Uncertainty, tS_R $\left(t\sqrt{\Sigma\,(S_{\bar{x},i}\,\theta_i)^2}\right)$	Composite Degrees of Freedom, ν_R [Note (1)]	Student's t	Total Absolute Uncertainty, U_R $\left(t\sqrt{\left(\frac{B_R}{2}\right)^2 + S_R^2}\right)$	Total Absolute Upper Uncertainty Limit, U_R^+ $(U_R + q_R)$	Total Absolute Lower Uncertainty Limit, U_R^- $(U_R - q_R)$
$\dot{m}$	Mass Flow rate	lbm/s	138.38	1.08	0	0.55	N/A	2	1.21	0.61	0.61

NOTE:
(1) See Appendix B for formula.

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SECTION 10 — EXAMPLES

10.1 FLOW MEASUREMENT USING PITOT TUBE

10.1.1 Define the Measurement Process. The flow rate of an incompressible fluid in a pipe may be determined by multiplying the integrated-average velocity of the fluid by the cross-sectional flow area of the pipe. One technique for measuring the integrated-average velocity of the fluid is to traverse the cross-sectional flow area with a pitot tube. Measurements at each traverse point can be used to determine local fluid velocity. Traverse points are typically specified at the centroid of equal areas so that the integrated-average velocity may be estimated as the average of the measured values for all traverse points. For this example, the velocity is measured at 40 unique traverse points (10 traverse positions along four equally spaced radii) corresponding to the centroid of equal areas as shown in Fig. 10.1. A total of 60 measurements is taken in succession at each traverse point once the pitot tube is positioned.

Several simplifying assumptions are made for this example:

(a) the pipe diameter is large compared to the pitot tube diameter such that blockage effects and wall interference effects can be neglected;

(b) the velocity pressure developed across the pitot tube is automatically measured by a differential pressure transmitter;

(c) the output of the differential pressure transmitter is measured and recorded by a computerized data acquisition system (DAS);

(d) the DAS computes velocity for each measurement by taking the square root of the output of the differential pressure transmitter, making the appropriate corrections for fluid density, and making the appropriate calibration corrections and unit conversions;

(e) the DAS automatically takes 60 readings at each traverse point and computes and records average values and standard deviations for the data collected at each traverse point;

(f) the pitot tube, differential pressure transmitter, and DAS are calibrated together as a system; and

(g) the flow rate (and hence the velocity profile) remains constant for the duration of the test.

10.1.2 Data Summary. The computerized data acquisition system is used to compute average values from the 60 measurements at each traverse point using Eq. (4.2). The resulting average values at each traverse point, $\overline{X}_{ij}$, are summarized in Table 10.1.

The traverse points are located at the centroid of equal areas so that the integrated-average velocity in the pipe is approximated by the average of the velocities determined at the traverse points. First, the average velocity along each radius, $\overline{V}_i$, is approximated as

$$\overline{V}_i\ (\text{ft/s}) \approx \sum_{j=1}^{10} \frac{\overline{X}_{ij}\ (\text{ft/s})}{10} \qquad (10.1)$$

Then, the average velocity in the pipe, $\overline{V}$, is approximated as

$$\overline{V}\ (\text{ft/s}) \approx \sum_{i=1}^{4} \frac{\overline{V}_i\ (\text{ft/s})}{4} \qquad (10.2)$$

The subscripts i and j are used in the previous equations to designate radius and traverse positions, respectively.

The results of these calculations are summarized in Table 10.2.

10.1.3 List Elemental Uncertainty Sources. The sources of uncertainty which are considered random in this simplified example are those causing variation in the 60 repeated measurements of velocity at each traverse point. The sources of uncertainty which are considered systematic in this simplified example are the uncertainty of the calibration of the instruments used to measure and record velocity at each traverse point, and the uncertainty of the integrated-average velocity due to spatial variation.

10.1.4 Calculate Random Standard Deviation. The DAS is programmed to output the standard deviation of the 60 measurements at each traverse point using

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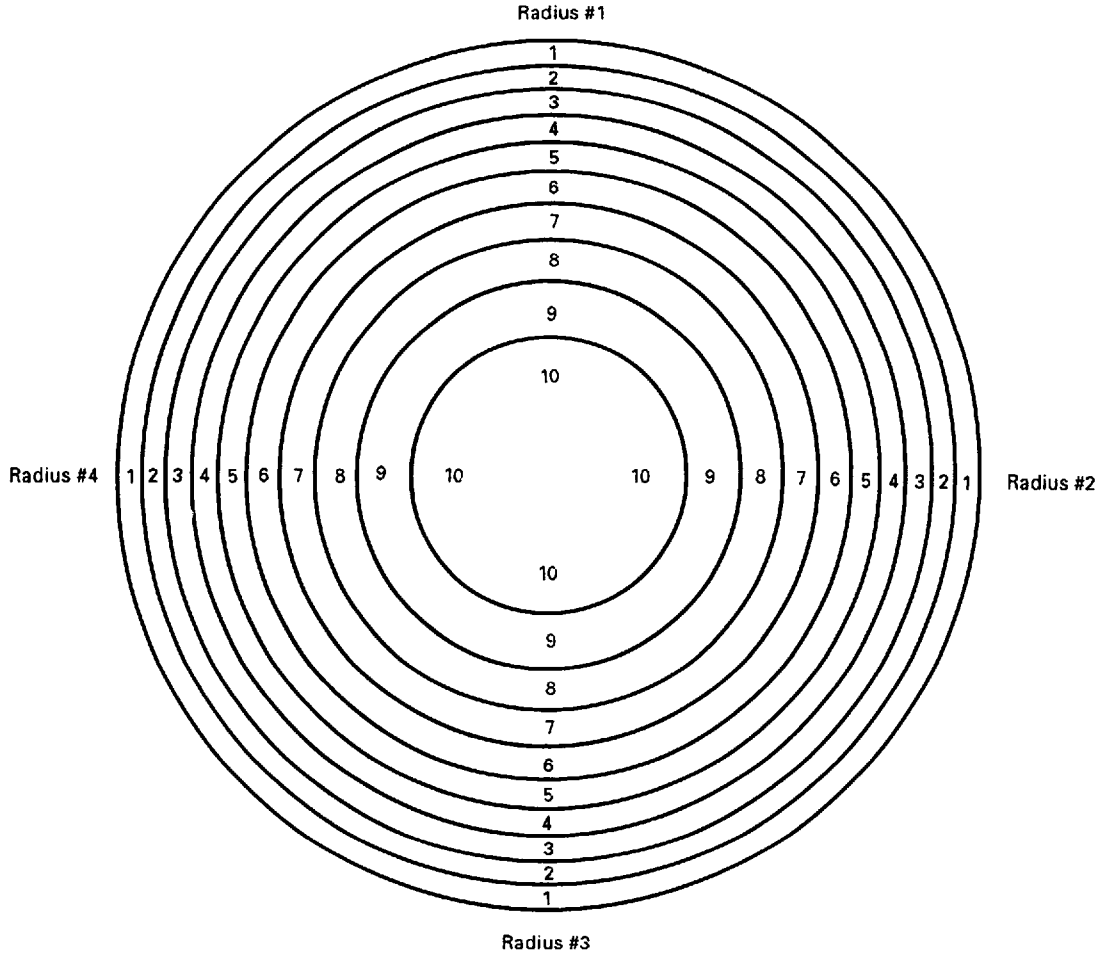

FIG. 10.1 TRAVERSE POINTS (EXAMPLE 10.1)

TABLE 10.1
AVERAGE VALUES (EXAMPLE 10.1)

Traverse Point	Radius 1, $i = 1$ $\overline{X}_{ij}$ (ft/s)	Radius 2, $i = 2$ $\overline{X}_{ij}$ (ft/s)	Radius 3, $i = 3$ $\overline{X}_{ij}$ (ft/s)	Radius 4, $i = 4$ $\overline{X}_{ij}$ (ft/s)
$j = 1$	5.31	5.27	5.21	5.00
$j = 2$	5.46	5.53	5.25	5.16
$j = 3$	5.55	5.61	5.37	5.31
$j = 4$	5.63	5.68	5.47	5.42
$j = 5$	5.65	5.74	5.58	5.50
$j = 6$	5.69	5.77	5.62	5.55
$j = 7$	5.73	5.79	5.65	5.63
$j = 8$	5.74	5.76	5.65	5.65
$j = 9$	5.76	5.75	5.70	5.65
$j = 10$	5.72	5.80	5.70	5.67

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TABLE 10.2
SUMMARY OF AVERAGE VELOCITY CALCULATION (EXAMPLE 10.1)

Parameter	Value
$\overline{V}_1$ (ft/s)	5.62
$\overline{V}_2$ (ft/s)	5.67
$\overline{V}_3$ (ft/s)	5.52
$\overline{V}_4$ (ft/s)	5.45
$\overline{V}$ (ft/s)	5.57

TABLE 10.3
STANDARD DEVIATIONS (EXAMPLE 10.1)

Traverse Point	Radius 1, $i = 1$ $S_{x_{ij}}$ (ft/s)	Radius 2, $i = 2$ $S_{x_{ij}}$ (ft/s)	Radius 3, $i = 3$ $S_{x_{ij}}$ (ft/s)	Radius 4, $i = 4$ $S_{x_{ij}}$ (ft/s)
$j = 1$	1.21	1.31	1.61	1.41
$j = 2$	1.06	1.61	1.78	1.65
$j = 3$	1.03	1.36	1.89	1.26
$j = 4$	1.21	1.31	1.84	1.80
$j = 5$	1.29	1.06	1.65	2.04
$j = 6$	1.09	1.26	1.09	1.74
$j = 7$	0.81	1.03	1.43	1.61
$j = 8$	1.00	0.93	1.18	2.14
$j = 9$	1.15	1.34	1.36	1.43
$j = 10$	0.81	1.45	1.00	1.54

Eq. (4.1). The standard deviation at each point, $S_{x_{ij}}$, is summarized in Table 10.3.

Since there are 60 measurements at each traverse point, $N_{ij} = 60$, the degrees of freedom at each traverse point is

$$\nu_{ij} = N_{ij} - 1 = 59 \tag{10.3}$$

The standard deviation for each average velocity along a radius, $S_{\overline{V}_i}$, is calculated from Eq. (7.7) as

$$S_{\overline{V}_i} = \left[\sum_{j=1}^{10} \left(\frac{\partial \overline{V}_i}{\partial \overline{X}_{ij}} \frac{S_{X_{ij}}}{\sqrt{60}}\right)^2\right]^{1/2} \tag{10.4}$$

where

$$\frac{\partial \overline{V}}{\partial \overline{X}_{ij}} = \frac{1}{10}$$

The standard deviation of the average velocity in the pipe, $S_{\overline{V}}$, is then calculated as

$$S_{\overline{V}} = \left[\sum_{i=1}^{4} \left(\frac{\partial \overline{V}}{\partial \overline{V}_i} S_{\overline{V}_i}\right)^2\right]^{1/2} \tag{10.5}$$

where

$$\frac{\partial \overline{V}}{\partial \overline{V}_i} = \frac{1}{4}$$

The results are summarized in Table 10-4.

Note that the standard deviation of the average velocity in the pipe is influenced by the random errors in the velocity measurements and actual variations in the velocity with time for the 60 measurements at each traverse point. These values are not influenced by variation of velocity as a function of spatial position.

10.1.5 Calculate Systematic Uncertainties

10.1.5.1 Calibration. Velocity is measured at each point with the same pitot tube, digital pressure transmitter, and DAS. The pitot tube, digital pressure

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TABLE 10.4
STANDARD DEVIATION OF AVERAGE VELOCITY (EXAMPLE 10.1)

Parameter	Value
$S_{\overline{V}_1}$ (ft/s)	0.0440
$S_{\overline{V}_2}$ (ft/s)	0.0523
$S_{\overline{V}_3}$ (ft/s)	0.0618
$S_{\overline{V}_4}$ (ft/s)	0.0687
$S_{\overline{V}}$ (ft/s)	0.03

TABLE 10.5
UNCERTAINTY DUE TO SPATIAL VARIATION (EXAMPLE 10.1)

Parameter	Value
S_S (ft/s)	0.10
L	4
B_S (ft/s)	0.10

transmitter, and DAS are calibrated together as a system. For this example, the calibration uncertainty of the instruments is 3% of measured velocity. Since the same instrumentation is used for all measurements, the systematic uncertainties due to calibration for all traverse points are correlated. Equation (8.2) can be used to show that the uncertainty in the average velocity measurement, $\overline{V}$, is

$$B_C = 3\% = 0.03 \times 5.57 \text{ (ft/s)} = 0.17 \text{ (ft/s)} \quad (10.6)$$

10.1.5.2 Spatial Variation. The true value being measured is the integrated-average of the velocity over the cross-sectional flow area. Averaging the velocities at the centroid of equal area points is a numerical approximation of this integrated-average. Even if exact values of velocity are known at each of the traverse points, the analyst must recognize that the numerical average may not equal the integrated-average value over the entire cross-sectional flow area. This is due to incomplete sampling of a profile which varies as a function of spatial position. Therefore, there is an inherent uncertainty in the method used to approximate the integrated-average velocity. This uncertainty of method is sometimes referred to as uncertainty due to spatial variation. Since the velocity profile remains fixed for the duration of the test, the spatial variation is a source of systematic uncertainty for the test result.

This uncertainty can be estimated in a variety of ways, including:

(a) special tests which provide independent knowledge of the velocity profile;

(b) special tests which compare the measurement technique to other techniques which yield the desired integrated-average;

(c) published reports which document the uncertainty of similar measurement techniques in similar measurement situations; and

(d) evaluation of the variation in test measurements as a function of spatial position.

For this example, the velocity profile is distorted due to the presence of flow disturbances upstream of the measurement location. Therefore, special tests were conducted to map the velocity profile along each of the four equally spaced radii. The results of these tests indicate that the average of the 10 traverse locations along each radius sufficiently characterizes the integrated-average value along each radius. However, since no additional pipe taps are available for testing, it is unknown whether the average of the traverses of the four radii sufficiently characterizes the integrated average around the pipe due to circumferential variations. Therefore, the uncertainty due to spatial variation is estimated by evaluation of the variation in the radial averages. Assuming that the four measured radial averages come from a population of values which are normally distributed, an estimate of the systematic uncertainty in the average velocity measurement due to spatial variation is

$$B_S \approx \frac{2\,S_S}{\sqrt{L}} \quad (10.7)$$

where

B_S = the systematic uncertainty due to spatial variation

S_S = the standard deviation of the radial averages [computed using Eq. (4.1) and the radial averages presented in Table 10.2]

L = the number of locations (radii) used in the computation of the average value

The calculation is summarized in Table 10.5.

10.1.5.3 Combining Elemental Systematic Uncertainties. The systematic uncertainties due to cali-

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TABLE 10.6
UNCERTAINTY OF MEASUREMENT (EXAMPLE 10.1)

Symbol	Description	Units	Calculated Value	B	$S_{\overline{V}}$	$\nu_{\overline{V}}$	U_{95}
$\overline{V}$	Average flow velocity	ft/s	5.57	0.20	0.03	>30	0.21

bration and spatial variation are combined using Eq. (6.8) as follows:

$$B = [B_C^2 + B_S^2]^{1/2} = 0.20 \text{ (ft/s)} \qquad (10.8)$$

10.1.6 Total Uncertainty of the Measurement. The total uncertainty of the measurement is determined using Eq. (6.10) as follows:

$$U_{95} = 2\sqrt{\left(\frac{B}{2}\right)^2 + (S_{\overline{V}})^2} = 0.21 \text{ (ft/s)} \qquad (10.9)$$

The uncertainty of the measurement is summarized in Table 10.6.

10.2 FLOW RATE UNCERTAINTY [12]

10.2.1 General Description. In this example, the test objective is to determine the flow of water using a 6 × 4 in. venturi (see Fig. 10.2) within an uncertainty of 0.5%. A pretest analysis is required to determine if an uncalibrated venturi could be used to satisfy the test objective, and, if not, whether calibration of the venturi would achieve the desired objective.

The clearest way to present the results of the steps in the uncertainty analysis is to develop a table in which the names, definitions, values, uncertainties, sensitivities, etc., are displayed for each of the variables required for the analysis. The table associated with the uncalibrated case is shown here first, as Table 10.7. The table has been developed in accordance with the step-by-step procedure of Section 9. The steps in the development of the table are as follows:

(a) Define Measurement Process. Flow rate can be calculated by making the measurements required to define the independent variables found in Eq. (10.10) (see Fluid Meters Research, ASME, 6th Edition, 1971, p. 54):

$$\dot{m} = \frac{0.099702C\, d^2 \sqrt{\rho h}}{\sqrt{1 - \left(\frac{d}{D}\right)^4}} \qquad (10.10)$$

The definitions and values for each of the measurements used in the calculation of the mass flow rate are displayed in the first four columns of Table 10.7.

Uncalibrated Venturi Case

(b), (c), (d) List Elemental Systematic Uncertainty Sources; Estimate Elemental Uncertainties; Calculate the Systematic and Random Uncertainties. The systematic and random uncertainties are determined from experience. The degrees of freedom associated with each of the uncertainty estimates is greater than 30. The systematic uncertainties (also known as bias limits) for each input parameter are shown in Table 10.7 displayed on an absolute basis. The standard deviation of the mean of the measurements taken is shown for each of the independent variables. These values are also expressed in absolute terms, so the units are the same as the units for the measured parameter.

(e) Propagate the Systematic and Random Uncertainties. The sensitivity of the result to each of the individual parameter uncertainties is calculated, either numerically or analytically, in accordance with Section 7.2.

A quick and easy way to numerically calculate the sensitivity coefficients of the independent parameters is to develop a table using a spreadsheet program on the personal computer. Table 10.8 shows the results obtained using such a spreadsheet, along with the formulas used in the spreadsheet.

As can be seen by looking at the formulas shown in Table 10.8, the same basic equation for the calculation of $\dot{m}$ was repeated seven times (once for each independent parameter, P_i) with P_i being replaced with $P_i + P_iQ_P$, where Q_P is the quantity by which P_i is to be perturbed. A perturbed $\dot{m}$ is therefore calculated for each independent parameter, and the baseline value of $\dot{m}$ is subtracted from each perturbed $\dot{m}$. Dividing the difference by the amount by which P_i was perturbed (P_iQ_P) provides the absolute sensitivity coefficient.

As noted in para. 7.2.1, the sensitivity coefficient can also be determined analytically, by finding the partial derivatives of the result with respect to each of the measured parameters. Table 10.9 shows the formulas for the partial derivatives for each of the measured parameters with respect to the calculated

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TABLE 10.7
UNCALIBRATED CASE (EXAMPLE 10.2)

Independent Parameters					
Symbol	Description	Units	Nominal Value	Absolute Systematic Uncertainty (B_i)	Absolute Standard Deviation of the Mean ($S_{\bar{x},i}$)
C	Discharge coefficient	...	0.964	0.0075	0
d	Throat diameter	Inches	4	0.001	0
D	Inlet diameter	Inches	6	0.002	0
ρ	Water density at 60°F [25]	lbm/ft^3	62.37	0.004	0.002
h	Differential pressure head across venturi (68°F)	in. H_2O	100	0.3	0.4

GENERAL NOTES:
(a) The systematic and random estimates for density are based on water temperature measurements having systematic and random uncertainties of 0.2°F and 0.1°F, respectively.
(b) The systematic uncertainty for the differential pressure head is assumed to be one-half the least count of the manometer scale.

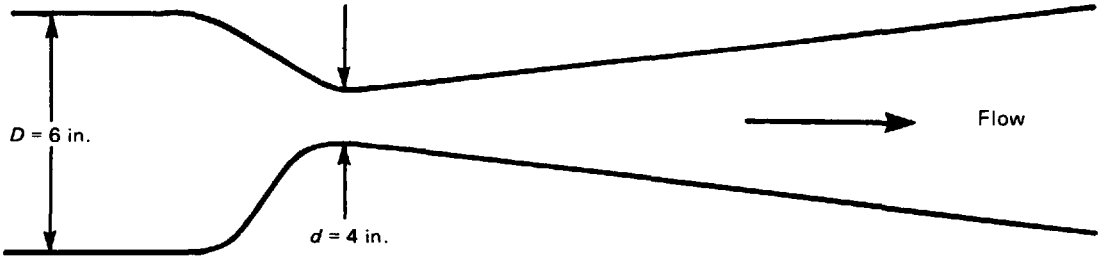

FIG. 10.2 SCHEMATIC OF A 6 × 4 in. VENTURI

mass flow rate, and the sensitivity coefficients found using these formulas.

The absolute contributions of the systematic and random uncertainties of the independent parameters are shown in the sixth and seventh columns of Table 10.10. The absolute contribution of the systematic uncertainties of each of the independent parameters to the uncertainty in the result (i.e., $\dot{m}$) are calculated by multiplying the sensitivity coefficient (θ) for each parameter by half of the systematic uncertainty (treating it as a standard deviation) of the parameter and squaring the product. The absolute contribution of the random uncertainties is found in a similar fashion, with the standard deviation of the mean of the measurements being multiplied by the sensitivity and the product squared. The square root of the sum of the systematic contributions (times Student's t which equals 2, as all degrees of freedom are greater than 30) yields the systematic uncertainty component of the result, and the random component of the result uncertainty is found by following the same procedure with the random contributions.

By looking at the numbers in the last two columns of Table 10.10, it can be seen at once which parameters contribute most to the uncertainty of the result. In this case, it is clear that the largest contributor is the systematic uncertainty in the discharge coefficient, C.

(f) Calculate Uncertainty. The total uncertainty of the result is then calculated by root-sum-squaring the systematic and random contributions. Table 10.11 shows the nominal value, and the systematic, random, and total uncertainties for $\dot{m}$, calculated as described above.

10.2.1.1 Relative Sensitivities and Uncertainties. Sensitivities and uncertainties may also be stated in relative terms. To convert the uncertainty of any parameter from absolute to relative terms, it is necessary only to divide the absolute uncertainty by the

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TABLE 10.8
RELATIVE SENSITIVITY COEFFICIENTS IN EXAMPLE 10.2 (CALCULATED NUMERICALLY)

Quantity by Which to Perturb Independent Parameters (Q_P): 0.001%			
Symbol	Nominal Values	Formulas for Absolute Sensitivity $\frac{\partial \dot{m}}{\partial P_i}$	Absolute Sensitivity $\frac{\partial \dot{m}}{\partial P_i}$
C	0.984	$\dfrac{\dfrac{0.099702\,(C + CQ_P)\,d^2\sqrt{\rho h}}{\sqrt{1-\left(\frac{d}{D}\right)^4}} - \dot{m}}{CQ_P}$	140
d	4	$\dfrac{\dfrac{0.099702C\,(d + dQ_P)^2\sqrt{\rho h}}{\sqrt{1-\left(\frac{d+dQ_P}{D}\right)^4}} - \dot{m}}{dQ_P}$	86.2
D	6	$\dfrac{\dfrac{0.099702C\,d^2\sqrt{\rho h}}{\sqrt{1-\left(\frac{d}{D+DQ_P}\right)^4}} - \dot{m}}{DQ_P}$	−11.4
ρ	62.37	$\dfrac{\dfrac{0.099702C\,d^2\sqrt{(\rho+\rho Q_P)h}}{\sqrt{1-\left(\frac{d}{D}\right)^4}} - \dot{m}}{\rho Q_P}$	1.11
h	100	$\dfrac{\dfrac{0.099702C\,d^2\sqrt{\rho\,(h+hQ_P)}}{\sqrt{1-\left(\frac{d}{D}\right)^4}} - \dot{m}}{hQ_P}$	0.692

Symbol	Result	Formula for $\dot{m}$
$\dot{m}$	138.386	$\dfrac{0.099702C\,d^2\sqrt{\rho h}}{\sqrt{1-\left(\frac{d}{D}\right)^4}}$

nominal value of the parameter. Multiplying by 100 would provide an expression of the uncertainty in terms of a percentage of the nominal value.

To convert the sensitivity coefficient to relative terms, the absolute sensitivity must be divided by the nominal value of the result and multiplied by the nominal value of the independent parameter. Multiplying by 100 would yield the percentage change in the result for a 1% change in the measured parameter. Tables 10.12 through 10.14 display the same parameters as Tables 10.6, 10.10, and 10.11, respectively, but with the sensitivities and uncertainties expressed in relative terms rather than absolute terms.

Calibrated Venturi Case

(a) Define the Measurement Process

(b), (c), (d) List Elemental Uncertainty Sources; Estimate Elemental Uncertainties; Calculate the Systematic and Random Uncertainties. In this case, the systematic uncertainty of the discharge coefficient changes. In addition, the systematic uncertainties of

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TABLE 10.9
ABSOLUTE SENSITIVITY COEFFICIENTS IN EXAMPLE 10.2 (CALCULATED ANALYTICALLY)

Symbol	Nominal Values	Formulas for Absolute Sensitivity $\frac{\partial \dot{m}}{\partial P_i}$	Absolute Sensitivity $\frac{\partial \dot{m}}{\partial P_i}$
C	0.984	$\dfrac{0.099702\ d^2\sqrt{\rho h}}{\sqrt{1-\left(\frac{d}{D}\right)^4}}$	140
d	4	$\sqrt{1-\left(\frac{d}{D}\right)^4}\ (2)\ 0.099702C\sqrt{\rho h}d - \left(\dfrac{(0.099702CYF\sqrt{\rho h})\ d^2\ (-4)\left(\frac{d}{D}\right)^3}{2\sqrt{1-\left(\frac{d}{D}\right)^4}\ D\left(1-\left(\frac{d}{D}\right)^4\right)}\right)$	86.2
D	6	$\dfrac{0.099702C\ d^2\sqrt{\rho h}\ 4d^4}{2\left(1-\left(\frac{d}{D}\right)^4\right)^{1.5} D^5}$	−11.4
ρ	62.37	$\dfrac{0.099702C\ d^2\ \frac{\sqrt{\frac{h}{\rho}}}{2}}{\sqrt{1-\left(\frac{d}{D}\right)^4}}$	1.11
h	100	$\dfrac{0.099702C\ d^2\ \frac{\sqrt{\frac{\rho}{h}}}{2}}{\sqrt{1-\left(\frac{d}{D}\right)^4}}$	0.692

the throat and pipe diameters are eliminated provided that the same dimensions which are used in the calibration process are also used in the test. By experience, for a calibrated venturi, the following estimate is used: $U_c = 0.122\%$.

(e), (f) Propagate the Systematic and Random Uncertainties; Calculate Uncertainty. These new values can be inserted into the tables (spreadsheets) used for the uncalibrated case. As the formulas do not change, there is nothing more that needs to be done. The parameters and results are displayed using a relative basis for the uncertainties and sensitivities in Tables 10.15 through 10.17.

It can be seen that the test objective of $\dot{m} \pm 0.5\%$ is marginally satisfied by using a calibrated venturi, and that the most promising path to take to obtaining additional reductions in the uncertainty of the mass flow rate determination would be through an increase in the number of differential head readings.

(g) Report. Tables 10.12 through 10.17 would be included in the report, with text noting that calibration

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TABLE 10.10
ABSOLUTE CONTRIBUTIONS OF SYSTEMATIC AND RANDOM UNCERTAINTIES OF INDEPENDENT PARAMETERS (EXAMPLE 10.2)

Independent Parameters						
Symbol	Description	Units	Nominal Value	Absolute Sensitivity (θ_i)	Absolute Systematic Uncertainty Contribution $\left(\frac{B_i}{2}\theta_i\right)^2$	Absolute Random Uncertainty Contribution $(S_{\bar{x},i}\theta_i)^2$
C	Discharge coefficient	...	0.964	140	2.78E-1	0
d	Throat diameter	Inches	4	86.2	1.86E-3	0
D	Inlet diameter	Inches	6	−11.4	1.29E-4	0
ρ	Water density at 60°F [25]	lbm/ft^3	62.37	1.11	4.92E-6	4.92E-6
h	Differential pressure head across venturi (at 68°F)	in. H_2O	100	0.692	1.08E-2	7.66E-2

TABLE 10.11
SUMMARY: NOMINAL VALUE; SYSTEMATIC, RANDOM, AND TOTAL UNCERTAINTIES IN ABSOLUTE TERMS FOR $\dot{m}$

Calculated Result							
Symbol	Description	Units	Calculated Value	Absolute Systematic Uncertainty, B_R $t\sqrt{\sum\left(\frac{B_i}{2}\theta_i\right)^2}$	Absolute Random Uncertainty, tS_R $t\sqrt{\sum(S_{\bar{x},i}\theta_i)^2}$	Student's t	Total Absolute Uncertainty, U_R $t\sqrt{\left(\frac{B_R}{2}\right)^2 + S_R^2}$
$\dot{m}$	Mass flow rate	lbm/s	138.38	1.08	0.55	2	1.21

TABLE 10.12
RELATIVE UNCERTAINTY OF MEASUREMENT (EXAMPLE 10.2)

Independent Parameters					
Symbol	Description	Units	Nominal Value	Relative Systematic Uncertainty (B_i)	Relative Standard Deviation of the Mean ($S_{\bar{x},i}$)
C	Discharge coefficient	...	0.984	0.762%	0%
d	Throat diameter	Inches	4	0.0250%	0%
D	Inlet diameter	Inches	6	0.0333%	0%
ρ	Water density at 60°F [25]	lbm/ft^3	62.37	0.0064%	0.0032%
h	Differential pressure head across venturi (at 68°F)	in. H_2O	100	0.300%	0.400%

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TABLE 10.13
RELATIVE CONTRIBUTIONS OF SYSTEMATIC AND RANDOM UNCERTAINTIES OF INDEPENDENT PARAMETERS (EXAMPLE 10.2)

Independent Parameters						
Symbol	Description	Units	Nominal Value	Relative Sensitivity (θ_i')	Relative Systematic Uncertainty Contribution $\left(\frac{B_i}{2}\theta_i'\right)^2$	Relative Random Uncertainty Contribution $(S_{\bar{x},i}\theta_i')^2$
C	Discharge coefficient	. . .	0.964	1.0000	1.45 E-0.5	0
d	Throat diameter	Inches	4	2.4923	9.71 E-0.8	0
D	Inlet diameter	Inches	6	−0.4923	6.73 E-0.9	0
ρ	Water density at 60°F [25]	lbm/ft^3	62.37	0.5000	2.57 E-10	2.57 E-10
h	Differential pressure head across venturi (at 68°F)	in. H_2O	100	0.5000	5.62 E-0.7	4.00 E-0.6

TABLE 10.14
SUMMARY: NOMINAL VALUE; SYSTEMATIC, RANDOM, AND TOTAL UNCERTAINTIES IN RELATIVE TERMS FOR THE UNCALIBRATED CASE (EXAMPLE 10.2)

Calculated Result							
Symbol	Description	Units	Calculated Value	Relative Systematic Uncertainty, B_R $t\sqrt{\sum\left(\frac{B_i}{2}\theta_i'\right)^2}$	Relative Random Uncertainty, tS_R $t\sqrt{\Sigma(S_{\bar{x},i}\theta_i')^2}$	Student's t	Total Relative Uncertainty, U_R $t\sqrt{\left(\frac{B_R}{2}\right)^2 + S_R^2}$
$\dot{m}$	Mass Flow rate	lbm/s	138.38	0.78%	0.40%	2	0.87%

TABLE 10.15
RELATIVE UNCERTAINTIES OF INDEPENDENT PARAMETERS (EXAMPLE 10.2)

Independent Parameters					
Symbol	Description	Units	Nominal Value	Relative Systematic Uncertainty (B_i)	Relative Standard Deviation of the Mean ($S_{\bar{x},i}$)
C	Discharge coefficient	. . .	0.964	0.122%	0%
d	Throat diameter	Inches	4	0.0250%	0%
D	Inlet diameter	Inches	6	0.0333%	0%
ρ	Water density at 60°F [25]	lbm/ft^3	62.37	0.0064%	0.0032%
h	Differential pressure head across venturi (at 68°F)	in. H_2O	100	0.300%	0.400%

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TABLE 10.16
RELATIVE CONTRIBUTIONS OF SYSTEMATIC AND RANDOM UNCERTAINTIES OF INDEPENDENT PARAMETERS (EXAMPLE 10.2)

Independent Parameters						
Symbol	Description	Units	Nominal Value	Relative Sensitivity (θ_i')	Relative Systematic Uncertainty Contribution $\left(\frac{B_i}{2}\theta_i'\right)^2$	Relative Random Uncertainty Contribution $(S_{\bar{x},i}\theta_i')^2$
C	Discharge coefficient	...	0.984	1.0000	3.72E-7	0
d	Throat diameter	Inches	4	2.49	9.71E-8	0
D	Inlet diameter	Inches	6	−0.492	6.73E-9	0
ρ	Water density at 60°F [25]	lbm/ft^3	62.37	0.500	2.57E-10	2.57E-10
h	Differential pressure head across venturi (at 68°F)	in. H_2O	100	0.500	5.62E-0.7	4.00E-0.6

TABLE 10.17
SUMMARY: NOMINAL VALUE; SYSTEMATIC, RANDOM, AND TOTAL UNCERTAINTIES IN RELATIVE TERMS FOR THE CALIBRATED CASE (EXAMPLE 10.2)

Calculated Result							
Symbol	Description	Units	Calculated Value	Relative Systematic Uncertainty, B_R $t\sqrt{\sum\left(\frac{B_i}{2}\theta_i'\right)^2}$	Relative Random Uncertainty, tS_R $t\sqrt{\Sigma(S_{\bar{x},i}\theta_i')^2}$	Student's t	Total Relative Uncertainty, U_R $t\sqrt{\left(\frac{B_R}{2}\right)^2 + S_R^2}$
$\dot{m}$	Mass flow rate	lbm/s	138.38	0.19%	0.40%	2	0.44%

of the venturi is required to meet the objective of 0.5% uncertainty or less in the mass flow rate determination.

10.3 FLOW RATE UNCERTAINTY INCLUDING NONSYMMETRIC DETERMINISTIC UNCERTAINTY

This example is nearly identical to that presented in para. 10.2 for the uncalibrated venturi case. The only differences are the following: (1) the discharge coefficient will have a nonsymmetric absolute systematic uncertainty represented by $B^- = 0.0095$ and $B^+ = 0.0055$, and (2) the differential pressure head across the venturi (at 68°F) will have a nonsymmetric absolute systematic uncertainty represented by $B^- = 0.1$ in. H_2O and $B^+ = 0.5$ in. H_2O. The results of this uncertainty analysis are presented in Table 10.18 in which each symbol has the same description as in Table 10.7, and in which each corresponding number has the same units as those given in Table 10.7.

Following the method given in para. 8.2, the absolute systematic uncertainty, B, for each variable is estimated by $B = (B^+ + B^-)/2$ and the apparent offset, q, is estimated by $q = (B^+ - B^-)/2$. The absolute sensitivity, θ, for each variable is estimated with all variables set to their offset values, $\bar{X} + q$. The apparent offset in the mass flow rate, q_r, is determined by the difference between the mass flow rate evaluated when all variables are set to their offset values, $R_{\bar{X}+q}$, and the mass flow rate evaluated when all variables are set to their measured values, $R_{\bar{X}}$. The lower and upper limits on the uncertainty of the mass flow rate are given by $U^- = U_{95,R} - q_r$, and $U^+ = U_{95,R} + q_r$, respectively.

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TABLE 10.18
RESULTS OF UNCERTAINTY ANALYSIS (EXAMPLE 10.3)

Symbol	$\bar{X}$	B^-	B^+	q	$\bar{X}+q$	B	$S_{\bar{X}}$	U_{95}	θ
C	0.984	0.0095	0.0055	−0.002	0.982	0.0075	0	0.0075	140.8
d	4	0.001	0.001	0	4	0.001	0	0.001	86.14
D	6	0.002	0.002	0	6	0.002	0	0.002	−11.34
ρ	62.37	0.004	0.004	0	62.37	0.004	0.002	0.0057	1.108
h	100	0.1	0.5	0.2	100.2	0.3	0.4	0.8544	0.6898
Symbol	$R_{\bar{X}}$	$R_{\bar{X}+q}$	q_R	$U_{95,R}$	U^-	U^+			
$\dot{m}$	138.38	138.24	−0.14	1.21	1.35	1.07			

TABLE 10.19
INPUT SYSTEMATIC AND RANDOM UNCERTAINTIES (EXAMPLE 10.4)

			In Measurement Units							
				Systematic Uncertainty [Note (1)]						
Symbol	Description	Measurement Units	Nominal Value	(B^+)	(B^-)	(q)	(B)	Random Uncertainty ($S_{\bar{X}}$)	Degrees of Freedom (ν)	Number of Measurements (N)
P_1	Inlet pressure	psia	14.70	...	...	...	0.02	0.03	>30	1
P_2	Exit pressure	psia	95.52	...	...	...	0.17	0.17	>30	1
T_1 [Note (2)]	Inlet temperature	°R	530.0	...	...	...	0.82	0.60	>30	1
T_2 [Note (2)]	Exit temperature	°R	960.0	...	...	...	1.63	0.70	>30	1
NOTES:										
(1) All the systematic uncertainties are assumed normally distributed and represented at 95% coverage.										
(2) Subset temperature uncertainties that root-sum-square to T_1 and T_2 uncertainties above are:										
For T_1										
	Thermocouple cal.	°R	...	...	...	...	0.50	0.20	>30	1
	Secondary ref.	°R	...	...	...	...	0.50	0.20	>30	1
	Ice ref.	°R	...	...	...	...	0.07	0.02	>30	1
	Data system	°R	...	...	...	...	0.40	0.50	>30	1
	RSS Total	°R	...	...	...	...	0.82	0.6	>30	...
For T_2										
	RSS* inlet	°R	...	...	...	...	0.82*	0.6	>30	1
	Probe cal.	°R	...	...	...	...	1.00	0.30	>30	1
	Cal. drift	°R	...	...	...	...	1.00	0.20	>30	1

*These uncertainties are correlated with those of the inlet temperatures.

10.4 COMPRESSOR PERFORMANCE UNCERTAINTY

10.4.1 General Description. In this example, the test objective is to measure air compressor performance by computing adiabatic efficiency. The measurements required are total pressure and total temperature at inlet and exit. A schematic of the locations of the required pressure and temperature measurements is shown in Fig. 10.3.

10.4.2 Determination of Uncertainty. According to the step-by-step procedure of Section 9, we have the following:

(a) Define Measurement Process. The functional relationship involved is

$$\eta = [(P_2/P_1)^{[(\gamma-1)/\gamma]} - 1]/[(T_2/T_1) - 1] \quad (10.11)$$

where

η = adiabatic compressor efficiency at η = 0.871
P_1 = inlet pressure, nominally 14.70 psia
P_2 = exit pressure, nominally 95.52 psia
T_1 = inlet temperature, nominally 530°R
T_2 = exit temperature, nominally 960°R
γ = ratio of specific heats, nominally 1.40

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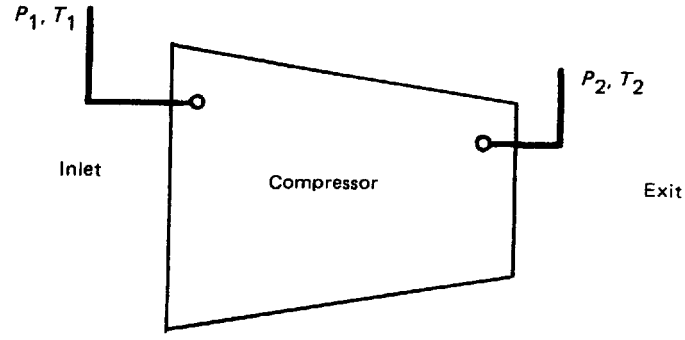

FIG. 10.3 TYPICAL PRESSURE AND TEMPERATURE LOCATIONS FOR COMPRESSOR EFFICIENCY DETERMINATION

(b), (c) List Elemental Error Sources; Calculate the Systematic and Random Uncertainties for Each Parameter. Table 10.19 lists typical systematic and random uncertainties for pressure and temperature.

(d) Propagate Systematic and Random Uncertainties. The individual parameter errors are propagated into efficiency units by a Taylor series expansion as given in Appendix C. The random uncertainty for efficiency is

$$S_{\bar{X},\eta} = [(\Theta_{P_1})^2 (S_{\bar{X},P_1})^2 + (\Theta_{P_2})^2 (S_{\bar{X},P_2})^2 + (\Theta_{T_1})^2 (S_{\bar{X},T_1})^2 + (\Theta_{T_2})^2 (S_{\bar{X},T_2})^2]^{1/2} \quad (10.12)$$

where the absolute sensitivities are

$$\Theta_{P_1} = \frac{-[(\gamma-1)/\gamma][(P_2/P_1)^{-1/\gamma}]\,[P_2/(P_1)^2]}{[(T_2/T_1)-1]} = -0.04090$$

$$\Theta_{P_2} = \frac{[(\gamma-1)/\gamma]\,[1/P_1]\,[(P_2/P_1)^{-1/\gamma}]}{[(T_2/T_1)-1]} = +0.00629$$

$$\Theta_{T_1} = \frac{[T_2]\,\{[(P_2/P_1)^{((\gamma-1)/\gamma)}]-1\}}{[(T_2-T_1)^2]} = +0.00367$$

$$\Theta_{T_2} = \frac{[-T_1]\,\{[(P_2/P_1)^{((\gamma-1)/\gamma)}]-1\}}{[(T_2-T_1)^2]} = -0.00203$$

The systematic uncertainty for efficiency is

$$B_\eta = [(\Theta_{T_1})^2 (B_{T_1})^2 + (\Theta_{T_2})^2 (B_{T_2})^2 + (\Theta_{P_1})^2 (B_{P_1})^2 + (\Theta_{P_2})^2 (B_{P_2})^2 + 2\,\Theta_{T_1}\,\Theta_{T_2}\,(B_{T_1T_2})]^{1/2} \quad (10.13)$$

where

$$B_{T_1T_2} = (0.5)\,(0.5) + (0.5)\,(0.5) + (0.07)\,(0.07) + (0.4)\,(0.4) = 0.665\ (°R)^2$$

$B_{T_1T_2}$ is the covariance of the error sources common to T_1 and T_2.

Note that in Eq. (10.13) an additional term has been added to account for the correlation between the inlet temperature systematic uncertainty B_{T_1} and that portion of the exit temperature systematic uncertainty B_{T_2} with which it correlates.

In this case, four of the elemental systematic uncertainties in B_{T_1} and B_{T_2} arise from identical error sources.

The absolute values of the sensitivities along with their corresponding systematic and random uncertainties are summarized in Table 10.20.

Utilizing Eqs. (10.12) and (10.13), the following expressions for the random standard deviation and the systematic uncertainty of the result are obtained by:

$$S_{\bar{X},\eta} = [(-0.04090)^2 (0.03)^2 + (0.00629)^2 (0.17)^2 + (0.00367)^2 (0.60)^2 + (-0.00203)^2 (0.70)^2]^{1/2} = [1.506 \times 10^{-6} + 1.143 \times 10^{-6} + 4.849 \times 10^{-6} + 2.019 \times 10^{-6}]^{1/2} \quad (10.14)$$

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TABLE 10.20
SYSTEMATIC AND RANDOM UNCERTAINTIES (EXAMPLE 10.4)

					In Result Units						
					Systematic Uncertainty						
Symbol	Description	Measurement Units	Nominal Value	Sensitivity (θ)	(B^+)	(B^-)	(q)	$\left[\theta\left(\frac{B}{2}\right)\right]^2$	Random Uncertainty $[\theta\,(S_{\bar{X}})]^2$	Degrees of Freedom (ν)	Number of Measurements (N)
P_1	Inlet pressure	psia	14.70	−0.04090	...	...	...	0.167 E-06	1.506 E-06	>30	1
P_2	Exit pressure	psia	95.52	0.00629	...	...	...	0.286 E-06	1.143 E-06	>30	1
T_1 [Note (1)]	Inlet temperature	°R	530.0	0.00367	...	...	...	2.26 E-06	4.849 E-06	>30	1
T_2 [Note (1)]	Exit temperature	°R	960.0	−0.00203	...	...	...	2.74 E-06	2.019 E-06	>30	1

NOTE:
(1) These systematic uncertainties have some components that are correlated (same uncertainty sources).

TABLE 10.21
RESULT SYSTEMATIC AND RANDOM UNCERTAINTIES (EXAMPLE 10.4)

			In Result Units							
				Systematic Uncertainty					Uncertainty	
Symbol	Description	Result Units	Calc. Result	($B^+/2$)	($B^-/2$)	Random Uncertainty ($S_{\bar{X},\eta}$)	Degrees of Freedom (ν)	Number of Measurements (N)	U_{95}^-	U_{95}^+
η	Compressor efficiency	Efficiency	0.8714	0.0015	0.0015	0.003	>30	1	0.0067	0.0067

$$S_{\bar{X},\eta} = 0.003 \text{ efficiency units} \qquad (10.15)$$

$$\begin{aligned} B_\eta/2 &= [(-0.04090)^2 \left(\frac{0.020}{2}\right)^2 + (0.00629)^2 \left(\frac{0.17}{2}\right)^2 \\ &\quad + (0.00367)^2 \left(\frac{0.82}{2}\right)^2 + (-0.00203)^2 \left(\frac{1.63}{2}\right)^2 \\ &\quad + (2)\,(0.00367)\,(-0.00203)\,(0.665/4)]^{1/2} \\ &= [0.167 \times 10^{-6} + 0.286 \times 10^{-6} + 2.26 \times 10^{-6} \\ &\quad + 2.74 \times 10^{-6} - 2.50 \times 10^{-6}]^{1/2} \qquad (10.16) \end{aligned}$$

$$B_\eta/2 = 0.0015 \text{ efficiency units} \qquad (10.17)$$

(e) Calculate Uncertainty. The uncertainty is obtained by combining the systematic uncertainty of Eq. (10.17) and the random standard deviation of Eq. (10.15). Because all the random uncertainties are based on experience, the degrees of freedom, ν, are assumed to be greater than 30 throughout. Therefore, using Welch-Satterthwaite would yield degrees of freedom greater than 30 for the uncertainty, thus Student's t is 2.00. The U_{95} uncertainty is calculated as

$$\begin{aligned} U_{95} &= \pm\,(2)\,[(B_\eta/2)^2 + (S_{\bar{X},\eta})^2]^{1/2} \\ &= \pm\,(2)\,[(0.0015)^2 + (0.003)^2]^{1/2} \\ &= \pm 0.0067 \text{ at 95\% coverage} \qquad (10.18) \end{aligned}$$

(f) Report. The final uncertainty report is shown in Table 10.21.

The calculation of uncertainty for the efficiency measurement is now complete. If it is desired to know the percent uncertainty, now is the time to calculate that. The efficiency level is 0.8714 ± 0.0067, or 0.88714 ± 0.77% for U_{95}.

When multiple measurements of the result are made (N times), the above $S_{\bar{X},\eta}$ value would need to be divided by $\sqrt{N}$. This accounts for the averaging of several measurements of the result.

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TABLE 10.22
SPECIFIED CONDITIONS (EXAMPLE 10.5)

Parameter	Specified Conditions
Throttle pressure, $P_1{}^c$	800 psig
Throttle temperature, $T_1{}^c$	850°F
Exhaust pressure, $P_2{}^c$	3.5 in. Hg
Generator output power factor, pF^c	85%
Generator hydrogen pressure, P_{H2}	30 psig
Generator size	100,000 kW

10.5 STEAM TURBINE PERFORMANCE UNCERTAINTY

10.5.1 General Description. One of the more complicated tests described by Performance Test Codes is that of a steam turbine. As an example of the uncertainty procedures for a turbine test, the steam rate test described in [12] is analyzed.

Note that the number of readings, the quality of test instrumentation, and the test data proper were assumed for the purpose of this example. The test described in this example is not in strict accordance with PTC 6.

The objective of the test is to determine the corrected steam rate of a nonextracting, condensing turbine used in a gas turbine–steam turbine combined cycle power plant. The corrected steam rate is determined by measuring the steam rate at test conditions, SR^t, and correcting the steam rate for specified conditions, SR^c. The specified conditions for the cycle are shown in Table 10.22. A simplified diagram of the nonextraction condensing turbine is shown in Fig. 10.4.

10.5.2 Define Measurement Process

10.5.2.1 Description of Test Instrumentation. The location of test instrumentation is shown in Fig. 10.4. Throttle steam pressure is measured at two locations with calibrated, digital absolute-pressure indicators. Throttle steam temperature is measured at two locations with calibrated, digital RTD temperature indicators. The exhaust pressure is measured at eight different locations in the exhaust casing with calibrated, absolute, digital pressure indicators connected to basket-type probes. The generator output power and power factor are determined with a calibrated, digital, three-phase power meter connected to potential and current transformers. Generator hydrogen pressure is measured with a calibrated Bourdon-tube pressure gage. Steam flow to the turbine is derived by measuring condensate flow using a calibrated throat-tap nozzle and correcting it for miscellaneous leakages and storages.

The goal of the test is to determine the sustained corrected steam rate for the cycle over a 1 hr period. Time variations in the process or instruments which occur over a time frame of greater than 1 hr are classified as systematic. Time variations in the process or instruments which occur over a time frame of less than 1 hr are classified as random. In light of the goal of the test and due to the relatively slow variation in most process variables, samples taken at a rate of greater than once per minute or sampled at more than one location each minute will be considered correlated (with respect to time) and will be averaged together and treated as a single data sample. The minimum sampling period is once per minute resulting in 60 one minute data samples. Calibration corrections are applied to all test data.

10.5.2.2 Summary of Test Data. The mean value of each measured parameter over the test duration is determined by averaging the 60 one minute data samples. The sample standard deviation associated with random uncertainty is determined by taking the standard deviation of the 60 one minute data samples for each parameter. The standard deviation of the mean value is determined by dividing the sample standard deviation by the square root of the number of samples. A summary of the test data is shown in Table 10.23.

10.5.2.3 Calculation of Steam Rate for Specified Conditions. The following calculation methodology is used to determine steam rate at the specified conditions based on measurement of the above parameters at test conditions. The calculation methodology and associated correction curves, constants, and assumptions were agreed upon by all parties prior to the test. In addition, the uncertainty of the calculation methodology, correction curves, con-

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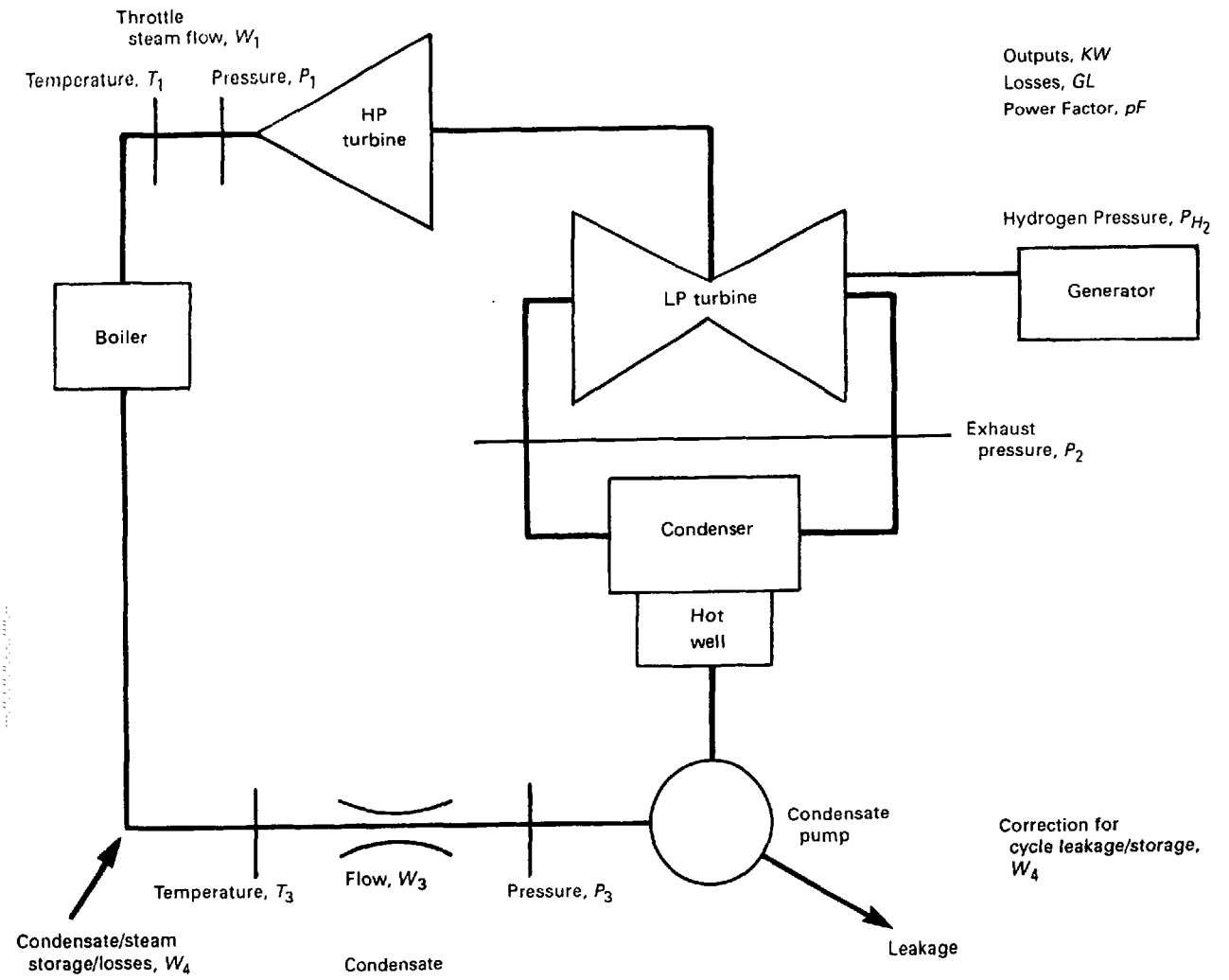

FIG. 10.4 SIMPLIFIED FLOW AND INSTRUMENTATION DIAGRAM FOR A NONEXTRACTION CONDENSING TURBINE

stants, and assumptions were excluded from consideration.

The throttle steam flow to the turbine is established from the measured condensate flow with corrections for any leakage from the cycle and any change in cycle storage between the throttle and the condensate flow nozzle.

$$w_1^t = w_3^t + w_4^t$$

where

w_1^t = steam flow at test conditions (lbm/hr)
w_3^t = measured condensate flow (lbm/hr)
w_4^t = correction for leakage/storage (lbm/hr)

The measured generator output is corrected for deviations from the specified values of power factor and hydrogen pressure using the generator loss curve, Fig. 10.5, supplied with the turbine performance data.

$$KW^c = KW^t + GL^t - GL^c$$

where

KW^c = corrected generator output (kW)
KW^t = measured generator output (kW)
GL^t = generator losses at test conditions (kW)

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TABLE 10.23
SUMMARY OF TEST DATA (EXAMPLE 10.5)

Parameter	Mean Value ($\bar{X}$)	Number of Samples (N)	Sample Standard Deviation (S_x)	Standard Deviation of the Mean Value ($S_{\bar{x}}$)
Throttle pressure, $P_1{}^t$ (psia)	813.7	60	5.2	0.67
Throttle temperature, $T_1{}^t$ (°F)	851.6	60	4.5	0.58
Exhaust pressure, $P_2{}^t$ (in. H_g)	3.38	60	0.23	0.03
Condensate flow, $w_3{}^t$ (lbm/hr)	892766	60	5000	645
Generator output power, KW^t (kW)	110131	60	500	65
Generator power factor, pF^t (%)	89.2	60	0.25	0.03
Generator hydrogen pressure, $P_{H2}{}^t$ (psig)	28.8	60	0.5	0.065
Cycle leakage/ storage, $w_4{}^t$ (lbm/ hr)	−76	60	2.5	0.32

GL^c = corrected generator losses (kW)

The test steam rate with specified generator hydrogen pressure and power factor is calculated by dividing the test value of turbine throttle flow by the corrected output:

$$SR^t = \frac{w_1{}^t}{KW^c}$$

where

SR^t = steam rate at test conditions (lbm/kWhr)

The throttle steam flow at specified conditions is

$$w_1{}^s = w_1{}^t \sqrt{\left(\frac{P_1{}^s}{P_1{}^t}\right)\left(\frac{\nu_1{}^t}{\nu_1{}^s}\right)}$$

where

$w_1{}^s$ = throttle steam flow at specified conditions (lbm/hr)

$P_1{}^s$ = throttle pressure at specified conditions (psia)

$P_1{}^t$ = throttle pressure at test conditions (psia)

$v_1{}^s$ = throttle specific volume at specified conditions (ft^3/lbm)

$v_1{}^t$ = throttle specific volume at test conditions (ft^3/lbm)

The test steam rate is corrected for deviations from the specified throttle steam and exhaust conditions by applying the correction divisors determined from Figs. 10.6, 10.7, and 10.8. The corrected value of the throttle steam flow, $w_1{}^s$, is used in determining those corrections which vary as a function of the steam flow.

$$SR^s = \frac{SR^t}{C_{P_1}\, C_{T_1}\, C_{P_2}}$$

where

SR^s = steam rate at specified conditions (lbm/kWhr)

C_{P_1} = throttle pressure correction factor

C_{T_1} = throttle temperature correction factor

C_{P_2} = exhaust pressure correction factor

10.5.3 Uncertainty of Measurements

10.5.3.1 Throttle Pressure. Throttle steam pressure is measured at two locations (90 deg. apart) with calibrated, digital, absolute-pressure indicators. The two locations provide for detection of spatial variation across the measurement plane. The measured throttle pressure is determined as the average value of the two locations reducing the effects of

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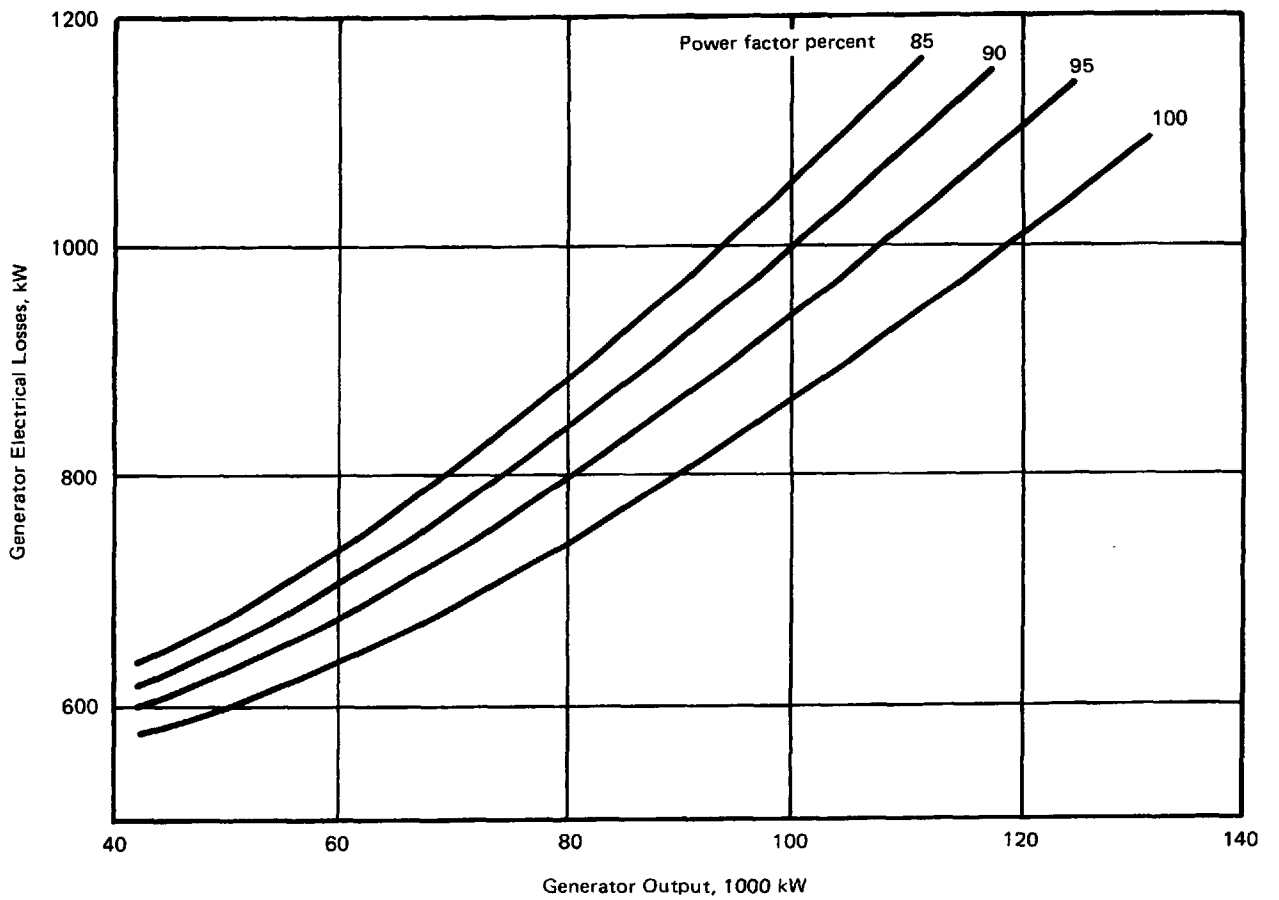

GENERAL NOTE:
Curves are for 30 psig Hydrogen pressure. For other Hydrogen pressures, correct curves at rate of 4.33 kW/psi pressure.

FIG. 10.5 GENERATOR ELECTRICAL LOSSES [12] (EXAMPLE 10.5)

spatial variation on the measurement. Analysis of the test data demonstrates no detectable systematic difference between these two locations.

The pressure indicators are of identical construction, are calibrated at the same facility, and are used in an identical manner. The systematic uncertainties associated with these elemental sources are considered equal and perfectly correlated for each pressure indicator. Since the systematic uncertainties of both pressure indicators are perfectly correlated, averaging the two readings does not reduce the systematic uncertainty. Instead, the systematic uncertainty of the average result equals the systematic uncertainty of a single instrument. The calibration histories for the indicators demonstrate a calibration uncertainty of ±0.1% of span. The calibration span is 0 to 1000 psia. The pressure indicators are installed in the same orientation as calibrated to avoid zero shift due to mounting position. The ambient temperature at the installation locations varies as much as ±20°F from the temperature at which the indicators were calibrated. According to the manufacturer's specifications, this can have an effect of up to ±0.1% of span.

The uncertainty due to random error is evaluated using the standard deviation of the sample mean shown in Table 10.23. The elemental uncertainty sources are summarized in Table 10.24. All elemental uncertainties are expressed in terms of psia.

10.5.3.2 Throttle Temperature. Throttle steam temperature is measured at two locations (90 deg. apart) with calibrated, digital, RTD temperature indicators. The two locations provide for detection of

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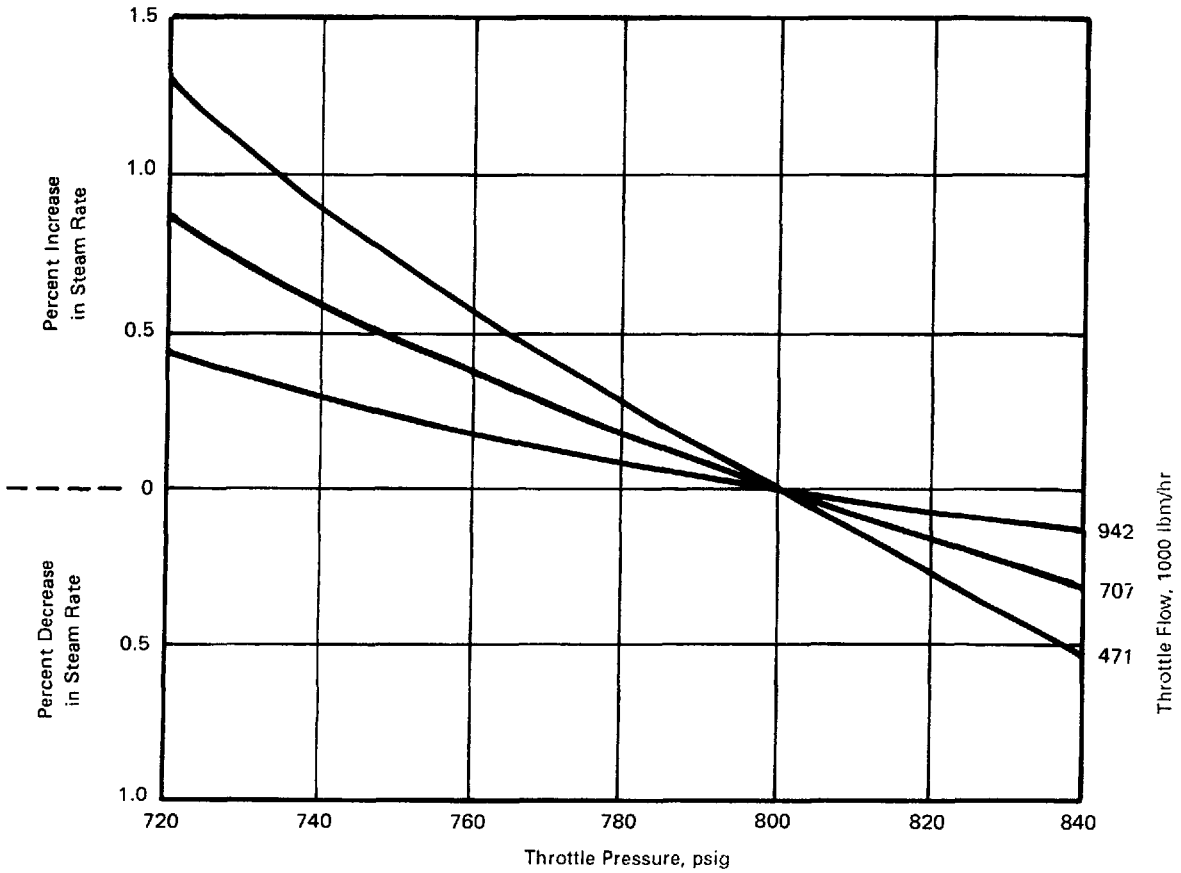

FIG. 10.6 THROTTLE PRESSURE CORRECTION [12] (EXAMPLE 10.5)

spatial variation across the measurement plane. The measured throttle temperature is determined as the average value of the two locations, thus reducing the effects of spatial variation on the measurement. Analysis of the test data demonstrates no detectable systematic difference between these two locations.

The RTD temperature indicators are of identical construction, are calibrated at the same facility, and are used in an identical manner. The systematic uncertainties associated with these elemental sources are considered equal and perfectly correlated for each temperature indicator. Since the systematic uncertainties of both temperature indicators are perfectly correlated, averaging the two readings does not reduce the systematic uncertainty. Instead, the systematic uncertainty of the average result equals the systematic uncertainty of a single instrument. The RTD indicator uses a four-wire, alternating current, low-level excitation measurement technique which minimizes uncertainties due to parasitic resistance, parasitic voltage, and self-heating. The RTD sensor and indicator are calibrated as a system to minimize calibration uncertainty. The calibration histories for the temperature indicators demonstrate a calibration uncertainty of ±1°F. The calibration span is 32 to 1000°F. The ambient temperature at the installation locations varies as much as ±20°F from the temperature at which the indicators were calibrated. According to the manufacturer's specifications, this can have an effect of up to ±0.5°F. The RTD sensors are installed in thermowells. The insertion lengths of the sensors and wells are sufficiently deep to minimize conduction error. The radiation error for the RTD sensors is estimated as ±0.5°F.

The uncertainty due to random error is evaluated using the standard deviation of the sample mean shown in Table 10.23. The elemental uncertainty sources are summarized in Table 10.25. All elemental uncertainties are expressed in terms of °F.

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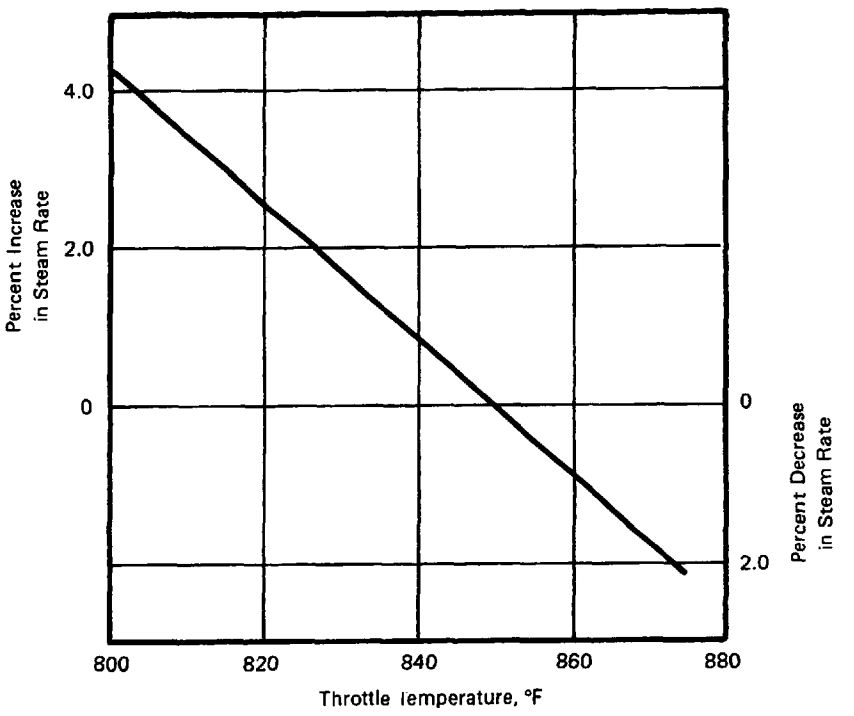

FIG. 10.7 THROTTLE TEMPERATURE CORRECTION [12] (EXAMPLE 10.5)

10.5.3.3 Exhaust Pressure. Exhaust pressure is measured with calibrated, digital, absolute-pressure indicators at eight locations using basket-type probes located one foot away from the wall of the exhaust annulus. The eight locations provide for determination of spatial variation across the measurement plane. The measured exhaust pressure is determined as the average value of the eight locations, thus reducing the effect of spatial variation on the measurement. Inspection of the test data demonstrates significant systematic differences between the locations. The systematic uncertainty due to the spatial variation is determined using the techniques prescribed in [13]. The systematic uncertainty due to spatial variation is estimated as ±0.08 in. Hg.

The pressure indicators are of identical construction, are calibrated at the same facility, and are used in an identical manner. The systematic uncertainties associated with these elemental sources are considered equal and perfectly correlated for each pressure indicator. Since the systematic uncertainties of the pressure indicators are perfectly correlated, averaging the readings does not reduce the systematic uncertainty. Instead, the systematic uncertainty of the average result equals the systematic uncertainty of a single instrument. The calibration histories for the indicators demonstrate a calibration uncertainty of ±0.1% of span. The calibration span is 0 to 30 in. Hg. The pressure indicators are installed in the same orientation as calibrated to avoid zero shift due to mounting position. The ambient temperature at the installation locations varies as much as ±20°F from the temperature at which the indicators were calibrated. According to the manufacturer's specifications, this can have an effect of up to ±0.1% of span.

The uncertainty due to random error is evaluated using the standard deviation of the sample mean shown in Table 10.23. The elemental uncertainty sources are summarized in Table 10.26. All elemental uncertainties are expressed in terms of in. Hg.

10.5.3.4 Condensate Flow. Reference [13] provides guidelines for evaluation of the systematic uncertainty of conventional flow sections used in performance testing. The guidelines are based on published and unpublished data comparing flow measurement using nozzles and orifices installed

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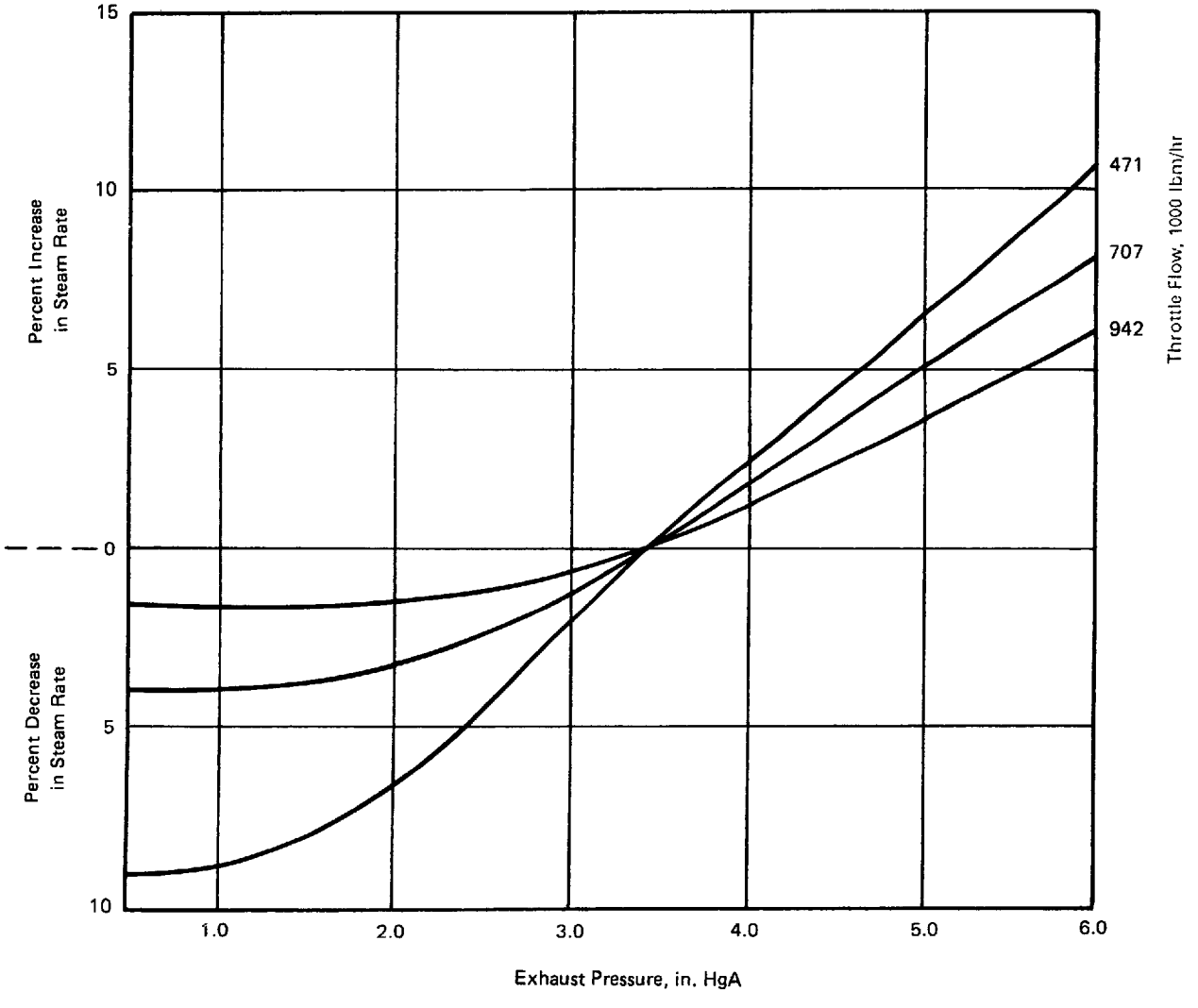

FIG. 10.8 EXHAUST PRESSURE CORRECTION [12] (EXAMPLE 10.5)

TABLE 10.24
THROTTLE PRESSURE ELEMENTAL UNCERTAINTIES (EXAMPLE 10.5)

Elemental Uncertainty Source	Systematic, B (psia)	Random, $S_{\bar{x}}$ (psia)	Degrees of Freedom, ν
Spatial variation	0	. . .	. . .
Calibration	1	. . .	. . .
Mounting effect	0	. . .	. . .
Ambient temperature effect	1	. . .	. . .
Random error	. . .	0.67	59
Total	1.4	0.67	59

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TABLE 10.25
THROTTLE TEMPERATURE ELEMENTAL UNCERTAINTIES (EXAMPLE 10.5)

Elemental Uncertainty Source	Systematic, B (°F)	Random, $S_{\bar{x}}$ (°F)	Degrees of Freedom, ν
Spatial variation	0	...	...
Parasitic resistance	0	...	...
Parasitic voltages	0	...	...
Self-heating error	0	...	...
Calibration	1	...	...
Ambient temperature effect	0.5	...	...
Conduction error	0	...	...
Radiation error	0.5	...	...
Random error	...	0.58	59
Total	1.2	0.58	59

TABLE 10.26
EXHAUST PRESSURE ELEMENTAL UNCERTAINTIES (EXAMPLE 10.5)

Elemental Uncertainty Source	Systematic, B (in. Hg)	$S_{\bar{x}}$ (in. Hg)	Degrees of Freedom, ν
Spatial variation	0.08	...	...
Calibration	0.03	...	...
Mounting effect	0	...	...
Ambient temperature effect	0.03	...	...
Random error	...	0.03 in. Hg	59
Total	0.09	0.03 in. Hg	59

under ideal conditions with those installed under typical conditions found in industry.

The condensate flow is measured using a throat-tap flow nozzle. Critical flow section specifications for the uncertainty evaluation are

(a) the flow element was calibrated before permanent installation and has been in service for less than 6 months;

(b) the flow element was not inspected immediately before or after the test;

(c) the flow nozzle β ratio is 0.5;

(d) the flow section has no flow straightener;

(e) the predominant flow disturbance preceding the flow element consists of a single 90 deg. bend seven diameters upstream of the nozzle; and

(f) the straight length of pipe downstream of the flow element is 3.5 diameters.

10.5.3.4.1 Base Uncertainty. The base uncertainty of the flow section is related to the flow element's initial calibration, time since last inspection, and service conditions. The base uncertainty is characterized under item E of Table 4.10 of [13], U_B= 2.5% of flow rate.

10.5.3.4.2 Uncertainty Due to Upstream Flow Disturbances. The flow coefficient for a conventional flow element is based on the response of the device to a fully developed flow profile. Locating the flow element in the presence of upstream flow disturbances introduces uncertainty in the flow measurement. Table 4.11 and Fig. 4.5 of [13] are used to estimate the uncertainty due to upstream flow disturbances. For a β ratio of 0.5 and a single elbow disturbance seven diameters upstream of the nozzle, U_{LNS} = 1.25% of flow rate.

10.5.3.4.3 Uncertainty Due to Downstream Flow Disturbances. The flow coefficient for a conventional flow element is based on the response of the device to a fully developed flow profile. Locating the flow element in the presence of downstream flow disturbances introduces uncertainty in the flow measurement. Table 4.11 and Fig. 4.9 of [13] are

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used to estimate the uncertainty due to downstream flow disturbances. For a downstream straight length of 3.5 diameters from the nozzle, U_{DSL} = 0.7% of flow rate.

10.5.3.4.4 Uncertainty Due to β Ratio. Fig. 4.6 of [13] presents additional uncertainty for the flow section based on the β ratio. The additional uncertainty for a calibrated flow nozzle with a β ratio of 0.5 is U_β = 0% of flow rate.

10.5.3.4.5 Uncertainty of Measurement Loop. The differential pressure induced across the condensate flow nozzle is measured by a digital, differential pressure indicator and converted to flow using the following simplified equation:

$$w_3{}^t = C\sqrt{\Delta P}$$

where

C= the composite flow coefficient for the flow nozzle at test conditions (lbm/hr/in. $H_2O^{1/2}$)

ΔP= the differential pressure measured across the flow nozzle taps (in. H_2O).

The relative sensitivity coefficient for flow with respect to differential pressure is

$$\theta' = \frac{\partial w/w}{\partial \Delta P/\Delta P} = 1/2 = \frac{\Delta P}{w}\left(\frac{\partial w}{\partial \Delta P}\right)$$

The differential pressure indicator is calibrated in place and zeroed at line pressure immediately prior to and after the test. In-place calibration and zeroing at line pressure minimizes uncertainties due to mounting position, ambient temperature effects, and static pressure effects. The uncertainty of the differential pressure transducer is characterized by ±0.1% of span. The span is 0 to 800 in. H_2O. At test conditions the differential pressure is 200 in. H_2O. The relative uncertainty of the pressure transducer is

$$\frac{U_{\Delta P}}{\Delta P} = \frac{(\pm 0.001)\ (800)}{200} = \pm 0.004$$

$$\frac{\text{in. } H_2O}{\text{in. } H_2O} \text{ or } \pm 0.4\% \text{ of pressure reading}$$

The resulting uncertainty in terms of flow is

$$\frac{U_w}{w} = \theta'\left(\frac{U_{\Delta P}}{\Delta P}\right) = (1/2)\ (\pm 0.004) = \pm 0.002$$

$$\frac{\text{lbm/hr}}{\text{lbm/hr}} \text{ or } \pm 0.2\% \text{ of flow reading}$$

The uncertainty due to random error is evaluated using the standard deviation of the sample mean shown in Table 10.23. The elemental uncertainty sources are summarized in Table 10.27. All elemental uncertainties are expressed in terms of lbm/hr.

10.5.3.5 Generator Output. Generator output power and power factor are measured by a digital, three-phase power meter connected to the generator via three-phase, four-wire connections. Use of a three-phase meter eliminates uncertainties in metering methods due to unbalanced loads. The calibration history of the power meter for the intended range of power and power factor demonstrates an uncertainty in power measurement within ±0.5% of reading and an uncertainty in power factor measurement within ±1%. The ambient temperature at the installation location varies as much as ±20°F from the temperature at which the power meter was calibrated. According to the manufacturer, the ambient temperature effect is ±0.5% of power reading. The power meter is connected to the generator via installed potential and current transformers. Type calibration curves are available for the potential and current transformers. The transformers are operated at or below burden volt-amperes and power factor. The uncertainty for the potential and current transformers is estimated from [13]. For potential transformers, the uncertainty is within ±0.3%. For current transformers, the uncertainty is within ±0.1%.

The uncertainty due to random error is evaluated using the standard deviation of the sample mean shown in Table 10.23. The elemental uncertainty sources are summarized in Table 10.28. All elemental uncertainties are expressed in terms of kW.

10.5.3.6 Generator Hydrogen Pressure. Generator hydrogen pressure is measured with a Bourdon tube pressure gage. The calibration history of the pressure gage demonstrates an uncertainty within ±0.5 psig.

The uncertainty due to random error is evaluated using the standard deviation of the sample mean shown in Table 10.23. The elemental uncertainty sources are summarized in Table 10.29. All elemental uncertainties are expressed in terms of psig.

10.5.3.7 Cycle Leakage/Storage. The correction for cycle leakage/storage is determined from mea-

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TABLE 10.27
CONDENSATE FLOW ELEMENTAL UNCERTAINTIES (EXAMPLE 10.5)

Elemental Uncertainty Source	Systematic, B (lbm/hr)	Random, $S_{\bar{x}}$ (lbm/hr)	Degrees of Freedom, ν
Base	22,000 (2.5%)	...	...
Upstream flow disturbances	11,000 (1.25%)	...	...
Downstream flow disturbances	6,200 (0.7%)	...	...
Beta ratio	0 (0%)	...	...
Measurement loop	1,800 (0.2%)	...	...
Random	...	645 (0.072%)	59
Total	25,000 (2.990%)	645 (0.072%)	59

TABLE 10.28
GENERATOR OUTPUT ELEMENTAL UNCERTAINTIES (EXAMPLE 10.5)

Elemental Uncertainty Source	Systematic, B (kW)	Random, $S_{\bar{x}}$ (kW)	Systematic, B (% pF)	Random, $S_{\bar{x}}$ (% pF)	Degrees of Freedom, ν
Power meter calibration	550 (0.5%)	...	0.5	...	...
Ambient temperature effect	550 (0.5%)	...	...	...	...
Potential transformers	330 (0.3%)	...	...	...	...
Current transformers	110 (0.1%)	...	...	...	...
Random	...	65	...	0.03	59
Total	852 (0.77%)	65	0.5	0.03	59

TABLE 10.29
GENERATOR HYDROGEN PRESSURE ELEMENTAL UNCERTAINTIES (EXAMPLE 10.5)

Elemental Uncertainty Source	Systematic, B (psig)	Random, $S_{\bar{x}}$ (psig)	Degrees of Freedom, ν
Pressure gage calibration	0.5	...	...
Random	...	0.065	59
Total	0.5	0.065	59

surements of any leakage from the cycle and any change in cycle storage between the throttle and the condensate flow nozzle.

The uncertainty due to random error is evaluated using the standard deviation of the sample mean shown in Table 10.23. The elemental uncertainty sources are summarized in Table 10.30. All elemental uncertainties are expressed in terms of lbm/hr.

10.5.3.8 Measurement Uncertainty Summary. The mean values and uncertainty components of each measured parameter are summarized in Table 10.31.

10.5.4 Uncertainty of Result

10.5.4.1 Sensitivity Analysis. The uncertainty of the test result (steam rate at specified conditions) is determined by propagating individual test uncertainties.

The uncertainty of each measured parameter is expressed in terms of the result using the propagation equations of paras. 7.3 and 7.4. The sensitivity coefficients required for propagation are determined numerically in accordance with para. 7.2. The numerical development of the sensitivity coefficients is presented in Table 10.32. First, the mean test

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TABLE 10.30
CYCLE LEAKAGE/STORAGE ELEMENTAL UNCERTAINTIES (EXAMPLE 10.5)

Elemental Uncertainty Source	Systematic, B (lbm/hr)	Random, $S_{\bar{x}}$ (lbm/hr)	Degrees of Freedom, ν
Systematic	7.6 (10%)	. . .	. . .
Random	. . .	0.32	59
Total	7.6 (10%)	0.32	59

TABLE 10.31
MEASUREMENT UNCERTAINTY SUMMARY (EXAMPLE 10.5)

Symbol	Description	Units	Mean Value ($\bar{X}$)	Systematic Uncertainty (B)	Standard Deviation of Mean ($S_{\bar{x}}$)	Degrees of Freedom (ν)	Number of Measurements (N)
$P_1{}^t$	Throttle pressure	psia	813.7	1.4	0.67	59	60
$T_1{}^t$	Throttle temperature	°F	851.6	1.2	0.58	59	60
$P_2{}^t$	Exhaust pressure	in. Hg	3.38	0.09	0.03	59	60
$w_4{}^t$	Condensate flow	lbm/hr	892,766	25,000	645	59	60
KW^t	Generator power	kW	110,131	852	65	59	60
pF^t	Generator power factor	%	89.2	0.5	0.03	59	60
$P_{H2}{}^t$	Generator hydrogen pressure	psig	28.8	0.5	0.065	59	60
$w_4{}^t$	Cycle leakage	lbm/hr	−76	7.6	0.32	59	60

result is calculated using the mean values of the measurements shown in Table 10.23 and the calculation methodology in para. 10.5.2.3. The test result is then recalculated for positive and negative incremental changes in each parameter. Each column in Table 10.32 represents a single result calculation for an incremental change in the designated parameter. The upper rows of the table show the parameter values used in each result calculation. The middle rows show intermediate steps in the calculation. The bottom rows show the calculation result and corresponding sensitivity coefficient. The sensitivity coefficient is determined as the change in the result divided by the incremental change in parameter value. Positive and negative incremental values show the degree of nonlinearity in the sensitivity around the test value.

Several practical considerations should be considered when performing numerical sensitivity analysis. The sensitivity coefficient, by definition, is the partial derivative of the result with respect to the parameter at the test point. Numerical determination of this derivative should be done with an increment size which is large enough to determine the difference in the result with sufficient precision, but small enough to best approximate the derivative. A process for achieving an acceptable increment size is to start with a large enough increment yielding a substantial change in the result and then gradually reduce the increment size until the change in the result converges for the positive and negative increments. A second consideration involves the use of performance curves for the sensitivity analysis. The readability of the performance curves is poor for small incremental values. Sufficiently large increments must be used consistent with the readability of the curves, or the slope of the curve at the test point should be approximated and used to determine small increments from the test point.

10.5.4.2 Propagation of Measurement Uncertainties. The measured parameter uncertainties are expressed in terms of the result using the propagation equations in paras. 7.3 and 7.4. The results are shown in Table 10.33.

10.5.4.3 Uncertainty of Result. The uncertainty of the result is determined in accordance with paras. 7.3, 7.4, and 7.5. The test result and test uncertainty are presented in Table 10.34.

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TABLE 10.32A
NUMERICAL DEVELOPMENT OF SENSITIVITY COEFFICIENTS (EXAMPLE 10.5)

Symbol	Units	Mean Value	P_1^t (+)	P_1^t (−)	T_1^t (+)	T_1^t (−)	P_2^t (+)	P_2^t (−)	w_3^t (+)
P_1^t	psia	813.7	834.7	794.7	813.7	813.7	813.7	813.7	813.7
T_1^t	°F	851.6	851.6	851.6	860.0	840.0	851.6	851.6	851.6
P_2^t	in. Hg	3.38	3.38	3.38	3.38	3.38	4.00	3.00	3.38
w_3^t	lbm/hr	892,766	892,766	892,766	892,766	892,766	892,766	892,766	901,694
KW^t	kW	110,131	110,131	110,131	110,131	110,131	110,131	110,131	110,131
pF^t	%	89.2	89.2	89.2	89.2	89.2	89.2	89.2	89.2
P_{H2}^t	psig	28.8	28.8	28.8	28.8	28.8	28.8	28.8	28.8
w_4^t	lbm/hr	−76	−76	−76	−76	−76	−76	−76	−76
w_1^t	lbm/hr	892,690	892,690	892,690	892,690	892,690	892,690	892,690	901,618
GL^t	kW	1,088	1,088	1,088	1,088	1,088	1,088	1,088	1,088
GL^c	kW	1,147	1,147	1,147	1,147	1,147	1,147	1,147	1,147
KW^c	kW	110,072	110,072	110,072	110,072	110,072	110,072	110,072	110,072
SR^t	lbm/kWhr	8.110	8.110	8.110	8.110	8.110	8.110	8.110	8.191
P_1^s	psia	814.7	814.7	814.7	814.7	814.7	814.7	814.7	814.7
v_1^t	ft³/lbm	0.9056	0.8814	0.9283	0.9128	0.8957	0.9056	0.9056	0.9056
v_1^s	ft³/lbm	0.9031	0.9031	0.9031	0.9031	0.9031	0.9031	0.9031	0.9031
w_1^s	lbm/hr	894,474	871,270	916,377	898,023	889,571	894,474	894,474	903,419
C_{P1}		1.0000	0.9990	1.0010	1.0000	1.0000	1.0000	1.0000	1.0000
C_{T1}		0.9986	0.9986	0.9986	0.9912	1.0088	0.9986	0.9986	0.9986
C_{P2}		0.9970	0.9970	0.9970	0.9970	0.9970	1.0103	0.9888	0.9970
SR^s	lbm/kWhr	8.146	8.154	8.138	8.207	8.063	8.039	8.213	8.227
ΔSR^s	lbm/kWhr		0.008	−0.008	0.061	−0.082	−0.107	0.068	0.081
ΔP	psi		21	−19	8.40	−11.60	0.62	−0.38	8928
$\Delta SR^s/\Delta P$	(lbm/kWhr)/ psi		0.00039	0.00043	0.0072	0.0071	−0.17	−0.18	9.1E-06

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TABLE 10.32B
NUMERICAL DEVELOPMENT OF SENSITIVITY COEFFICIENTS (EXAMPLE 10.5)

Symbol	w_3^t (−)	KW^t (+)	KW^t (−)	pF^t (+)	pF^t (−)	$P_{H2}{}^t$ (+)	$P_{H2}{}^t$ (−)	w_4^t (+)	w_4^t (−)
P_1^t	813.7	813.7	813.7	813.7	813.7	813.7	813.7	813.7	813.7
T_1^t	851.6	851.6	851.6	851.6	851.6	851.6	851.6	851.6	851.6
P_2^t	3.38	3.38	3.38	3.38	3.38	3.38	3.38	3.38	3.38
w_3^t	883,838	892,766	892,766	892,766	892,766	892,766	892,766	892,766	892,766
KW^t	110,131	110,682	109,580	110,131	110,131	110,131	110,131	110,131	110,131
pF^t	89.2	89.2	89.2	90.2	88.2	89.2	89.2	89.2	89.2
$P_{H2}{}^t$	28.8	28.8	28.8	28.8	28.8	29.8	27.8	28.8	28.8
w_4^t	−76	−76	−76	−76	−76	−76	−76	−75	−77
w_1^t	883,762	892,690	892,690	892,690	892,690	892,690	892,690	892,691	892,689
Cl^t	1,088	1,093	1,083	1,075	1,134	1,092	1,084	1,088	1,088
Cl^C	1,147	1,156	1,138	1,101	1,160	1,147	1,147	1,147	1,147
KW^c	110,072	110,619	109,525	110,105	110,105	110,076	110,068	110,072	110,072
SR^t	8.029	8.070	8.151	8.108	8.108	8.110	8.110	8.110	8.110
P_1^s	814.7	814.7	814.7	814.7	814.7	814.7	814.7	814.7	814.7
v_1^t	0.9056	0.9056	0.9056	0.9056	0.9056	0.9056	0.9056	0.9056	0.9056
v_1^s	0.9031	0.9031	0.9031	0.9031	0.9031	0.9031	0.9031	0.9031	0.9031
w_1^s	885,528	894,474	894,474	894,474	894,474	894,474	894,474	894,475	894,473
C_{P1}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
C_{T1}	0.9986	0.9986	0.9986	0.9986	0.9986	1.9986	0.9986	0.9986	0.9986
C_{P2}	0.9970	0.9970	0.9970	0.9970	0.9970	0.9970	1.9970	0.9970	0.9970
SR^s	8.064	8.106	8.187	8.143	8.143	8.146	8.146	8.146	8.146
ΔSR^s	−0.081	−0.040	0.041	−0.002	−0.002	−0.000	0.000	0.000	−0.000
ΔP	−8928	551	−551	1	−1	1	−1	0.76	−0.76
$\Delta SR^s/\Delta P$	9.1E-06	−7.3E-05	−7.4E-05	−0.0024	0.0024	−0.00030	−0.00030	9.1E-06	9.1E-06

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TABLE 10.33
PROPAGATION OF MEASUREMENT UNCERTAINTIES (EXAMPLE 10.5)

Symbol	Description	Mean Value ($\bar{x}$)	Sensitivity Coefficient (θ)	θB (lbm/kW hr)	$\theta S_{\bar{x}}$ (lbm/kW hr)	Degrees of Freedom (ν)	Number of Measurements (N)
P_1^t	Throttle pressure	813.7	0.00041	0.001	0.000	59	60
T_1^t	Throttle temperature	851.6	0.0072	0.009	0.004	59	60
P_2^t	Exhaust pressure	3.38	0.18	0.016	0.005	59	60
w_4^t	Condensate flow	892,766	9.1E-06	0.228	0.006	59	60
KW^t	Generator power	110,131	7.4E-05	0.063	0.005	59	60
pF^t	Generator power factor	89.2	0.0024	0.001	0.000	59	60
P_{H2}^t	Generator hydrogen pressure	28.8	0.00030	0.000	0.000	59	60
w_4^t	Cycle leakage/storage	−76	9.1E-06	0.000	0.000	59	60

TABLE 10.34
UNCERTAINTY OF RESULT (EXAMPLE 10.5)

Name	SR^c
Description	Steam Rate Corrected to Specified Conditions
Units	lbm/kWhr
Mean value	8.146
B_R [Eq. (7.12)]	0.237
S_R [Eq. (7.7)]	0.010
Degrees of freedom, ν_R	>30
Uncertainty, U_{R95} [Eq. (7.14)]	0.238 (2.92% of steam rate)

TABLE 10.35
PUMP DESIGN CURVE DATA WITH T_C = 20°C (EXAMPLE 10.6)

Flow			Differential Pressure	
m^3/s	gpm	Percent Rated Flow	kPa	psi
0.000	0	0	827	119.9
0.126	2000	100	689	99.9
0.189	3000	150	552	80.1

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TABLE 10.36
TEST RESULTS (EXAMPLE 10.6)

	Factory Test [Note (1)]	Field Tests				
	1/5/90	6/8/90	6/1/91	6/12/92	6/4/93	6/7/94
Raw data						
flow, m^3/s	0.126	0.125	0.126	0.13	0.123	0.129
P exit, Pa	...	840000	845000	820000	836000	841000
d exit, m	...	0.254	0.254	0.254	0.254	0.254
z exit, m	...	2.40	2.40	2.40	2.40	2.40
z inlet, m	...	15.00	15.00	15.00	15.00	15.00
T, °C	20.15	19.71	20.01	20.31	20.78	21.10
Resultants						
density, kg/m^3	997.7	997.8	997.7	997.7	997.6	997.5
ΔP, Pa	712,000	718,000	724,800	707,300	710,200	725,400
ΔP, psi	103.3	104.1	105.1	102.6	103.0	105.4

NOTE:
(1) The factory test data only provides resultant information.

It should be noted that the uncertainty is almost entirely dominated by the uncertainty of the condensate flow measurement. Reduction in the overall test uncertainty requires reduction in the uncertainty of this measurement.

10.6 COMPARATIVE PERIODIC TESTING

10.6.1 Problem Definition. Periodic testing of equipment is a common situation where measurement uncertainty must be considered. For this example, a pump is considered to perform consistently if the supply pressure measured at 100% of rated flow is consistent with prior test results. The design data for the pump are presented in Table 10.35. The test data for this example are presented in Table 10.36. After each test it is necessary to evaluate the test results. As part of this effort the effect of measurement uncertainty must be considered. For the first test in Table 10.36, the available information is very limited since it was a factory test (see Figs. 10.9 and 10.11).

The performance of a hydraulic system can be evaluated using Bernoulli's equation:

$$H = \frac{p}{\rho g} + \frac{v^2}{2g} + z$$

where
H = total head (m)
p = system pressure (Pa)
ρ = fluid density (kg/m^3)
v = fluid velocity (m/s)
g = gravity constant (9.81 m/s^2)
z = elevation (m)

The measure of pump performance is considered the effective head produced at a specified pressure. The effective head in terms of the measured test parameters and physical property data would be

$$\Delta p = p_2 + \frac{8Q^2\rho}{\pi^2 d_2^4} + (z_2 - z_1)\rho g$$

where
Δp = pressure increase between upstream and downstream pressure taps (Pa)
p_2 = exit pressure (Pa)
Q = fluid flow rate (m^3/s)
d_2 = inside pipe diameter at downstream pressure tap (m)
z_2 = elevation of downstream pressure tap (m)
z_1 = elevation of upstream pressure tap or free surface elevation or reservoir (m)

The density can be estimated numerically [14]:

$$\rho = 766.17 + 1.80396\,T_K - \frac{3.4589\,T_K^2}{1000} \qquad (10.19)$$

where

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$$T_K = T_C + 273.15$$

and T_C is the fluid temperature in °C.

Estimates of the density, ρ, using Eq. (10.19) are reported to have a standard deviation of 0.5868 kg/m^3. A reasonable systematic uncertainty estimate for Eq. (10.19) would then be

$$B_{\rho,\text{curve}} = 2.0\left(0.5868\ \frac{\text{kg}}{\text{m}^3}\right) = 1.2\ \frac{\text{kg}}{\text{m}^3}$$

Since the curve fit will always provide the same result for a specified temperature, the random uncertainty is considered zero.

During most tests it is not feasible to operate exactly at a specified operating condition. In the case of a pump test, the applied flow might be slightly different for each test. This will result in a slight change in the resultant differential pressure. The variation in flow may be handled by normalizing the test results, adding an additional random term. A normalizing technique will be used for this example. A normalization coefficient can be estimated from the factory test data in Table 10.35. The best fit for this data is

$$\Delta p = 827{,}000 - 376{,}000\ Q - 5{,}710{,}000\ Q^2 \quad (10.20)$$

The normalizing coefficient would be the slope of Eq. (10.20) at the specified test conditions.

$$\Delta p_N = b\,(Q_N - Q) + \Delta p \quad (10.21)$$

where

Δp_N = expected pressure increase based on the nominal (specified) test conditions (Pa)
Δp = the measured pressure increase (Pa)
Q_N = the nominal (specified) test flow (m^3/s)
Q = the measured test flow (m^3/s)
b = slope of Eq. (10.20)

The slope of Eq. (10.20) is

$$\left.\frac{\partial \Delta p_N}{\partial Q}\right|_{Q_N} = b = -376{,}000 - 11{,}420{,}000\ Q_N$$

or, since $Q_N = 0.126$ m^3/s,

$$b = -1{,}810{,}000\ \frac{\text{Pa} \cdot \text{s}}{\text{m}^3}$$

The final data reduction equation would then be

$$\Delta p = p_2 + \frac{8Q^2\rho}{\pi^2\, d_2^{\,4}} + (z_2 - z_1)\,\rho g + b\,(Q_N - Q)$$

10.6.2 Comparison With Independent Control. The field test data for the pump can be compared with the factory test results or the minimum rated pressure output. For this type of evaluation the uncertainties are independent and a simple comparison of the test results with the benchmark value is adequate. The uncertainty is calculated using the method in paras. 7.1 through 7.4.

The partial derivatives necessary to estimate the sensitivity coefficients for this problem are

$$\frac{\partial \Delta p}{\partial p_2} = 1$$

$$\frac{\partial \Delta p}{\partial Q} = \frac{16Q\rho}{\pi^2\, d_2^{\,4}} - b$$

$$\frac{\partial \Delta p}{\partial d_2} = \frac{-32Q^2\rho}{\pi^2\, d_2^{\,5}}$$

$$\frac{\partial \Delta p}{\partial \rho} = \frac{8Q^2}{\pi^2\, d_2^{\,4}} + (z_2 - z_1)\,g$$

$$\frac{\partial \Delta p}{\partial z_2} = \rho g$$

$$\frac{\partial \Delta p}{\partial z_1} = -\rho g$$

$$\frac{\partial \Delta p}{\partial b} = Q_N - Q$$

$$\frac{\partial \rho}{\partial T_C} = 1.80396 - \frac{6.9178\ T_K}{1000}$$

The sensitivity coefficient for temperature must be computed using a chain rule:

$$\frac{\partial \Delta p_N}{\partial T_C} = \frac{\partial \Delta p_N}{\partial \rho}\,\frac{\partial \rho}{\partial T_C} = \left(\frac{8Q^2}{\pi^2 d_2^{\,4}} + (z_2 - z_1)\,g\right)\left(1.80396 - \frac{6.9178\ T_K}{1000}\right)$$

The uncertainty for each test can be calculated as shown in Table 10.38. The uncertainties for each test are presented in Table 10.37. The results are plotted in Fig. 10.11. The test conclusions that can be deduced from this figure are

(a) the pump is operating consistently when compared to the factory test results since the uncertainty bands overlap;

(b) the pump is operating better than the minimum required design condition; the confidence for this conclusion is better than 95%.

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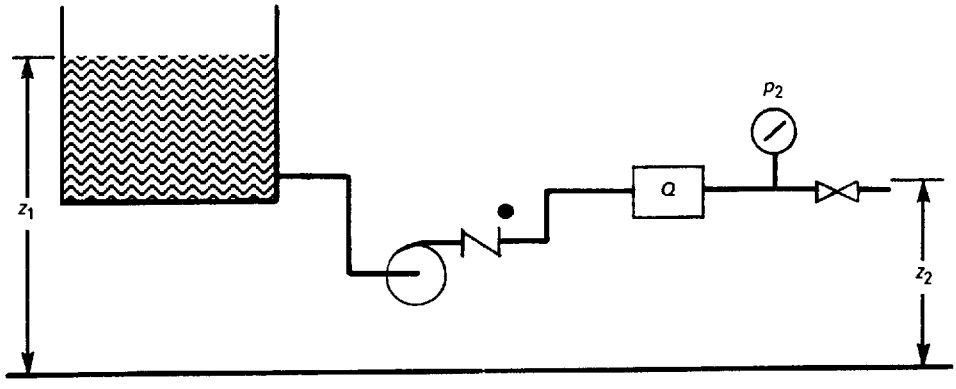

FIG. 10.9 INSTALLED ARRANGEMENT (EXAMPLE 10.6)

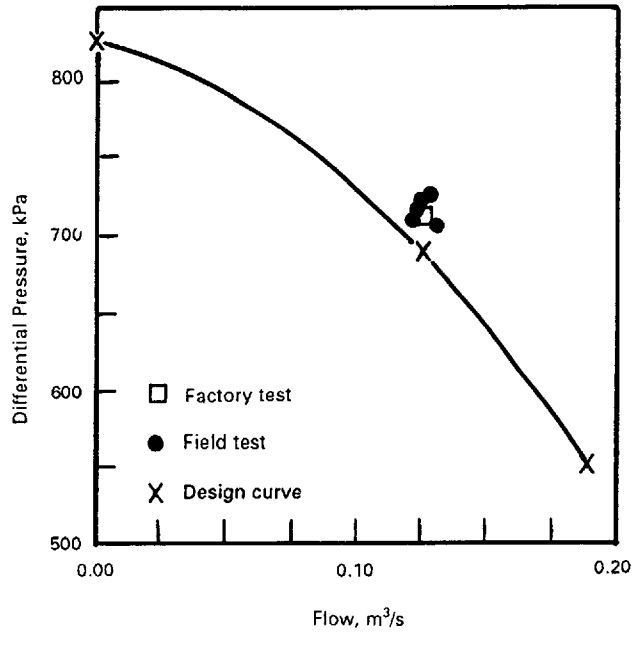

FIG. 10.10 PUMP DESIGN CURVE WITH FACTORY AND FIELD TEST DATA SHOWN (EXAMPLE 10.6)

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TABLE 10.37
TEST RESULTS (EXAMPLE 10.6)

Date	Δp kPa	B kPa	$S_{\bar{x}}$ kPa	U kPa	η	B_η	S_η	U_η
1/5/90	712	...	...	...	0.9916	0.0182	0.0049	0.0207
6/8/90	718	13	4	15	1.0000	0.0096	0.0069	0.0168
6/1/91	725	13	4	15	1.0097	0.0097	0.0070	0.0170
6/12/92	707	13	4	15	0.9847	0.0097	0.0069	0.0169
6/4/93	710	13	4	15	0.9889	0.0096	0.0069	0.0168
6/7/94	726	13	4	15	1.0111	0.0097	0.0070	0.0170

TABLE 10.38
UNCERTAINTY PROPAGATION FOR COMPARISON WITH INDEPENDENT CONTROL (EXAMPLE 10.6)

		Value	θ_i	B_i	$\theta_i B_i$	$S_{\bar{x},i}$	$\theta_i S_{\bar{x},i}$
Flow, m^3/s	Q	0.125	1.863e6	0.006	11178	0.001	1863.0
Exit pressure, Pa	p_2	840e3	1	7000	7000	3000	3000.0
Exit diameter, m	d_2	0.254	−47810	0.002	−96	0.0	−0.0
Exit elevation, m	z_2	2.40	9786	0.025	245	0.0	0.0
Inlet elevation, m	z_1	15.00	−9786	0.025	−245	0.0	−0.0
Fluid temperature, °C	T_C	19.71	26.76	0.5	13	0.1	3.0
Fluid density, m^3/s	ρ	997.8	−120.5	1.2	−145	0.0	−0.0
Correction coefficient	b	−1.815e6	0.0010	10000	10	0.0	0.0
Differential pressure, Pa	Δp	718000			13195		3541

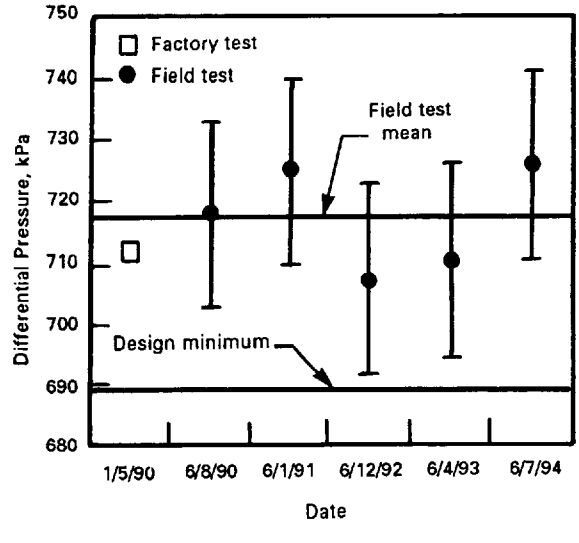

FIG. 10.11 COMPARISON OF TEST RESULTS WITH INDEPENDENT CONTROL CONDITION (EXAMPLE 10.6)

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10.6.3 Comparative Uncertainties. Correlation of terms is an important consideration in comparison testing [15, 16] where two different operating conditions or constructions are being compared by use of a ratio:

$$\eta = \frac{\Psi_{alt}}{\Psi_{control}} \qquad (10.22)$$

where

η = ratio of two resultants
Ψ_{alt} = alternate (or variable) resultant
$\Psi_{control}$ = resultant used for the baseline or control

For the pump test on 12 June 1992, with the 8 June 1990 test considered the control test, Eq. (10.22) becomes

$$\eta_{(6/12/92)} = \frac{707 \text{ kPa}}{718 \text{ kPa}} = 0.985$$

The test results for the comparative analysis are summarized in Table 10.37.

The uncertainty for the comparative analysis can be computed using the method from para. 8.1. The partial derivatives for Eq. (10.22) are

$$\frac{\partial \eta}{\partial \Psi_a} = \frac{\eta}{\Psi_a} \qquad \frac{\partial \eta}{\partial \Psi_c} = \frac{-\eta}{\Psi_c}$$

These may be combined with the partial derivatives presented earlier to estimate the sensitivity coefficients for the comparative analysis.

$$\theta_{p_{2,a}} = \frac{\eta}{\Delta p_a}\frac{\partial \Delta p_a}{\partial p_{2,a}} \quad \theta_{Q_a} = \frac{\eta}{\Delta p_a}\frac{\partial \Delta p_a}{\partial Q_a} \quad \theta_{d_{2,a}} = \frac{\eta}{\Delta p_a}\frac{\partial \Delta p_a}{\partial d_{2,a}}$$

$$\theta_{\rho_a} = \frac{\eta}{\Delta p_a}\frac{\partial \Delta p_a}{\partial \rho_a} \quad \theta_{z_{2,a}} = \frac{\eta}{\Delta p_a}\frac{\partial \Delta p_a}{\partial z_{2,a}} \quad \theta_{z_{1,a}} = \frac{\eta}{\Delta p_a}\frac{\partial \Delta p_a}{\partial z_{1,a}}$$

$$\theta_{b_a} = \frac{\eta}{\Delta p_a}\frac{\partial \Delta p_a}{\partial b_a} \quad \theta_{T_{C,a}} = \frac{\eta}{\Delta p_a}\frac{\partial \Delta p_a}{\partial T_{C,a}}$$

$$\theta_{p_{2,c}} = \frac{-\eta}{\Delta p_c}\frac{\partial \Delta p_c}{\partial p_{2,c}} \quad \theta_{Q_c} = \frac{-\eta}{\Delta p_c}\frac{\partial \Delta p_c}{\partial Q_c} \quad \theta_{d_{2,c}} = \frac{-\eta}{\Delta p_c}\frac{\partial \Delta p_c}{\partial d_{2,c}}$$

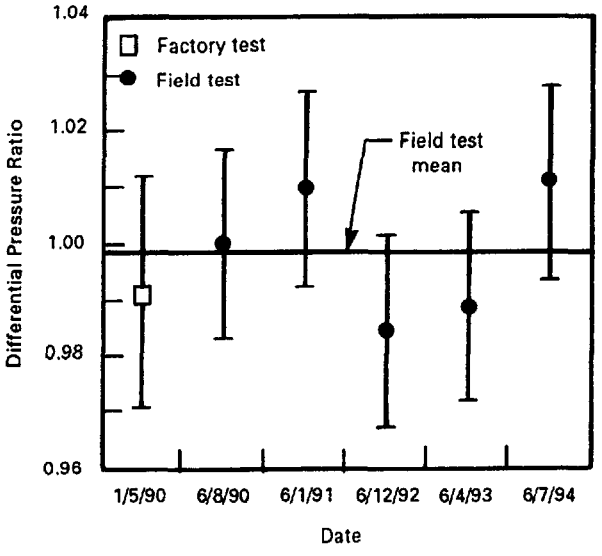

FIG. 10.12 COMPARISON OF TEST RESULTS USING THE INITIAL FIELD TEST AS THE CONTROL (EXAMPLE 10.6)

$$\theta_{\rho_c} = \frac{-\eta}{\Delta p_c}\frac{\partial \Delta p_c}{\partial \rho_c} \quad \theta_{z_{2,c}} = \frac{-\eta}{\Delta p_c}\frac{\partial \Delta p_c}{\partial z_{2,c}} \quad \theta_{z_{1,c}} = \frac{-\eta}{\Delta p_c}\frac{\partial \Delta p_c}{\partial z_{1,c}}$$

$$\theta_{b_c} = \frac{-\eta}{\Delta p_c}\frac{\partial \Delta p_c}{\partial b_c} \quad \theta_{T_{C,c}} = \frac{-\eta}{\Delta p_c}\frac{\partial \Delta p_c}{\partial T_{C,c}}$$

The uncertainty of the comparison ratio, η, when all of the systematic uncertainty terms are correlated and the corresponding sensitivity coefficients are equal (i.e., $\theta_{i,alt} = \theta_{i,control}$) is zero. For Eq. (10.22) the systematic uncertainty summation based on Eq. (8.2) is

$$\begin{aligned} B_\eta^2 = {} & \theta_{p_{2,a}}^2 B_{p_{2,a}}^2 + \theta_{Q_a}^2 B_{Q_a}^2 + \theta_{d_{2,a}}^2 B_{d_{2,a}}^2 \\ & + \theta_{\rho_a}^2 B_{\rho_a}^2 + \theta_{z_{2,a}}^2 B_{z_{2,a}}^2 + \theta_{z_{1,a}}^2 B_{z_{1,a}}^2 \\ & + \theta_{T_{C,a}}^2 B_{T_{C,a}}^2 + \theta_{b_a}^2 B_{b_a}^2 + \theta_{p_{2,c}}^2 B_{p_{2,c}}^2 \\ & + \theta_{Q_c}^2 B_{Q_c}^2 + \theta_{d_{2,c}}^2 B_{d_{2,c}}^2 + \theta_{\rho_c}^2 B_{\rho_c}^2 \\ & + \theta_{z_{2,c}}^2 B_{z_{2,c}}^2 + \theta_{z_{1,c}}^2 B_{z_{1,c}}^2 + \theta_{T_{C,c}}^2 B_{T_{C,c}}^2 \\ & + \theta_{b_c}^2 B_{b_c}^2 + 2\,\theta_{p_{2,a}}\theta_{p_{2,c}} B_{p_{2,a}p_{2,c}} \\ & + 2\,\theta_{Q_a}\theta_{Q_c} B_{Q_aQ_c} + 2\,\theta_{d_{2,a}}\theta_{d_{2,c}} B_{d_{2,a}d_{2,c}} \\ & + 2\,\theta_{\rho_a}\theta_{\rho_c} B_{\rho_a\rho_c} + 2\,\theta_{z_{2,a}}\theta_{z_{2,c}} B_{z_{2,a}z_{2,c}} \end{aligned}$$

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$$+ 2\,\theta_{z_{1,a}}\,\theta_{z_{1,c}}\,B_{z_{1,a}z_{1,c}} + 2\,\theta_{T_{C,a}}\,\theta_{T_{C,c}}\,B_{T_{C,a}T_{C,c}} + 2\,\theta_{b_a}\,\theta_{b_c}\,B_{b_ab_c}$$

For this example, all of the systematic uncertainties are considered fully correlated except that only some of the exit-pressure uncertainty sources are correlated. For the test on 12 June 1992, with the 8 June 1990 test considered the control test, the random component of the uncertainty $S_{\bar{x}}$ would be 0.0069 (see Table 10.39). The systematic uncertainty (from Tables 10.39 and 10.40) is

$$B = \sqrt{(0.0258)^2 + (-0.000572)} = 0.0097$$

The total comparative uncertainty is then

$$U = 2\sqrt{\left(\frac{0.0097}{2}\right)^2 + (0.0069)^2} = 0.0169$$

The uncertainty for each test can be calculated as shown in Tables 10.39 and 10.40. The uncertainties for each test are presented in Table 10.37. The results are plotted in Fig. 10.12. The test conclusion that can be drawn from this figure is the pump is operating consistently when compared since the uncertainty bands overlap.

TABLE 10.39
UNCERTAINTY PROPAGATION FOR COMPARATIVE UNCERTAINTY (EXAMPLE 10.6)

	$\frac{\partial \eta}{\partial \Delta p}$	$\frac{\partial \Delta p}{\partial x_i}$	θ_i	B_i	$\theta_i B_i$	$S_{\bar{x},i}$	$\theta_i S_{\bar{x},i}$
Variable test (6/12/92)							
Q	1.39E-6	1.863E-6	2.590	0.006	0.0155	0.001	0.0026
p_2	1.39E-6	1	1.39E-6	7000	0.0097	3000	0.0042
d_2	1.39E-6	−47810	−0.0665	0.002	−0.0001	0.0	−0.0000
z_2	1.39E-6	9786	0.01360	0.025	0.0003	0.0	0.0000
z_1	1.39E-6	−9786	−0.0136	0.025	−0.0003	0.0	−0.0000
T_C	1.39E-6	26.76	37.2E-3	0.5	0.0000	0.1	0.0000
ρ	1.39E-6	−120.5	−168E-3	1.2	−0.0002	0.0	−0.0000
b	1.39E-6	0.0010	1.39E-9	10000	0.0001	0.0	0.0000
Control test (6/8/90)							
Q	−1.37E-6	1.863E-6	−255.2	0.006	−0.0153	0.001	−0.0026
p_2	−1.37E-6	1	−1.37E-6	7000	−0.0096	3000	−0.0041
d_2	−1.37E-6	−47810	65.5E-3	0.002	0.0001	0.0	0.0000
z_2	−1.37E-6	9786	−13.4E-3	0.025	−0.0003	0.0	−0.0000
z_1	−1.37E-6	−9786	13.4E-3	0.025	0.0003	0.0	0.0000
T_C	−1.37E-6	26.76	−36.6E-6	0.5	−0.0000	0.1	−0.0000
ρ	−1.36E-6	−120.5	165E-6	1.2	0.0002	0.0	0.0000
b	−1.37E-6	0.0010	−1.37E-9	10000	−0.0000	0.0	−0.0000
η (RSS value)					0.0258		0.0069

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TABLE 10.40
SENSITIVITY COEFFICIENT ESTIMATES FOR COMPARATIVE ANALYSIS
(EXAMPLE 10.6)

	$\theta_{i,a}$	$B_{i,a}$	$\theta_{i,c}$	$B_{i,c}$	$2\,\theta_{i,a}\,\theta_{i,c}\,B_{i,ai,c}$
Q	2.593	0.006	−2.552	0.006	−0.000476
p_2	1.39E-3	5000	−1.37E-6	5000	−0.000095
d_2	−0.0719	0.002	65.5E-3	0.002	−0.000000
z_2	0.01360	0.025	−13.4E-3	0.025	−0.000000
z_1	−0.0136	0.025	13.4E-3	0.025	−0.000000
T_C	37.8E-6	0.5	−36.7E-6	0.5	−0.000000
ρ	−168E-6	1.2	165E-6	1.2	−0.000000
b	5.56E-9	10000	−1.37E-9	10000	−0.000000
η (sum)					−0.000572

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APPENDIX A — STATISTICAL CONSIDERATIONS

(This Appendix is not a part of ASME PTC 19.1-1998 and is included for information purposes only.)

A.1 UNDERSTANDING STATISTICAL INTERVALS

It is often desirable to use the terms *confidence, tolerance,* and *prediction interval.* (The use of *tolerance* in this Appendix is different from that of PTC 1.) An eloquent treatment of these terms is given by Gerald Hahn [17] in his 1970 paper, which is reproduced here by permission.

Statistical intervals are frequently misunderstood and misused. Here is an explanation of when to use confidence, tolerance, and prediction intervals.

> Engineers have come to appreciate that few things in life are known exactly. The most they can do is obtain an estimate and construct an interval which, with a high probability, contains the quantity of interest. This article describes three different types of statistical intervals and shows where each should be used.
>
> The three intervals are: (1) a confidence interval to contain a population mean, (2) a tolerance interval to contain a specified proportion of the population, and (3) a prediction interval to contain all of a specified number of future observations.
>
> Many nonstatistical users of statistics are well acquainted with confidence intervals. Some are also aware of tolerance intervals, but most nonstatisticians know very little about prediction intervals despite their practical importance. A frequent mistake is to calculate a confidence interval on the population mean when the actual problem calls for a tolerance interval or a prediction interval. At other times, a tolerance interval is used when a prediction interval is needed.
>
> This confusion is understandable since most texts on statistics devote extensive space to confidence intervals on population parameters, make limited reference to tolerance intervals, and almost never talk about prediction intervals. This is unfortunate because tolerance intervals or prediction intervals are needed as frequently in industrial applications as confidence intervals, and given the required tabulations, the procedure for constructing them is no more difficult. Table A.1 lists the information needed to construct all three intervals.

Confidence Interval for the Population Mean. The sample mean $\bar{y}$ is an estimate of the unknown mean μ, but differs from it because of sampling fluctuations. However, it is possible to construct a statistical interval known as a confidence interval for the population mean μ. This interval contains μ with a specific probability. This probability is known as the associated confidence level. Thus a 95 percent confidence interval on the population mean is an interval which contains μ with a probability of 0.95. It is calculated as

$$\bar{y} \pm c_M(n)\, s,$$

where $c_M{}^{(n)}$ is obtained from the first column of Table A.1 as a function of n, the sample size. For the example shown in the box $c_M(5) = 1.24$ and the 95 percent confidence interval for μ is:

$$50.10 \pm (1.24)(1.31).$$

Consequently, one can be 95 percent confident that the interval 48.48 to 51.72 contains the unknown value of μ. More precisely, over a large number of samples, the interval

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TABLE A.1
FACTORS FOR CALCULATING THE TWO-SIDED 95% PROBABILITY INTERVALS FOR A NORMAL DISTRIBUTION

Number of Given Observations	Factors for Confidence Interval to Contain the Population Mean μ	Factors for Tolerance Interval to Contain at Least 90%, 95% and 99% of the Population			Factors for Prediction Interval to Contain the Values of All of 1, 2, 5, 10, and 20 Future Observations				
n	$c_M(n)$	$c_{T,90}(n)$	$c_{T,95}(n)$	$c_{T,99}(n)$	$c_{P,1}(n)$	$c_{P,2}(n)$	$c_{P,5}(n)$	$c_{P,10}(n)$	$c_{P,20}(n)$
4	1.59	5.37	6.37	8.30	3.56	4.41	5.56	6.41	7.21
5	1.24	4.28	5.08	6.63	3.04	3.70	4.58	5.23	5.85
6	1.05	3.71	4.41	5.78	2.78	3.33	4.08	4.63	5.16
7	0.92	3.37	4.01	5.25	2.62	3.11	3.77	4.26	4.74
8	0.84	3.14	3.73	4.89	2.51	2.97	3.57	4.02	4.46
9	0.77	2.97	3.53	4.63	2.43	2.86	3.43	3.85	4.26
10	0.72	2.84	3.38	4.43	2.37	2.79	3.32	3.72	4.10
11	0.67	2.74	3.26	4.28	2.33	2.72	3.24	3.62	3.98
12	0.64	2.66	3.16	4.15	2.29	2.68	3.17	3.53	3.89
15	0.55	2.48	2.95	3.88	2.22	2.57	3.03	3.36	3.69
20	0.47	2.31	2.75	3.62	2.14	2.48	2.90	3.21	3.50
25	0.41	2.21	2.63	3.46	2.10	2.43	2.83	3.12	3.40
30	0.37	2.14	2.55	3.35	2.08	2.39	2.78	3.06	3.33
40	0.32	2.05	2.45	3.21	2.05	2.35	2.73	2.99	3.25
60	0.26	1.96	2.33	3.07	2.02	2.31	2.67	2.93	3.17
∞	0	1.64	1.96	2.58	1.96	2.24	2.57	2.80	3.02

A two-sided 95 percent interval is $\bar{y} \pm c(n)s$, where $c(n)$ is the appropriate tabulated value and $\bar{y}$ and s are the mean and the standard deviation of the given sample of size n.

calculated in this manner will contain the unknown mean 95 percent of the time.

Tolerance Interval to Contain a Specific Proportion of the Population. Instead of, or in addition to, a confidence interval to contain μ, many applications require an interval to enclose a specific proportion of the population. For a normal distribution, if μ and σ are known exactly, it can be stated that 90 percent of the population is located in the interval.

$$\mu \pm 1.64\sigma$$

However, if only sample estimates $\bar{y}$ and s of the population values μ and σ are given, the best that can be stated is that with a chosen probability (say 0.95) the interval contains at least 90, 95, or 99 percent of the population. Such an interval is called a tolerance interval and can be calculated for a normal population with the help of the factors $c_{T,90}(n)$, $c_{T,95}(n)$, and $c_{T,99}(n)$, shown in columns 2, 3, and 4 of Table A.1.

For example, it can be stated with 95 percent confidence that the interval:

$$\bar{y} \pm c_{T,90}(n)s$$

contains at least 90 percent of a normal population:

The tolerance interval for the example in the box at right may be calculated as

$$50.10 \pm (4.28)(1.31)$$

or 44.49 to 55.71 where $c_{T,90}(n)s = 4.28$. Thus, one may be 95 percent confident that the preceding interval contains at least 90 percent of the sampled population.

The fact that both a population proportion (or percentage) and a statistical probability (also a percentage) are associated with a tolerance interval is sometimes confusing to the engineer. The first of these numbers refers to the proportion (or percentage) of the population that the interval is to contain. The second number specifies the probability

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that the calculated interval really contains at least the specified proportion of the population. When μ and σ are known exactly, an interval to contain a specified proportion of the population may still be of interest, but, in this case, there is no longer any uncertainty associated with the proportion of the population contained in the interval.

Prediction Interval to Contain all of a Specified Number of Future Observations. Another type of interval is one that will contain all the values of one or more future observations. This is known as a prediction interval. The last five columns of Table A.1 provide values of the factor $c_{P,k}(n)$ such that all of k future observations from the same normal population will be located in the interval:

$$\bar{y} \pm c_{P,k}(n)\, s,$$

with a probability of 0.95.

For example, if two additional readings are taken from the example in the box, $k = 2$ and $n = 5$. From Table A.1 the factor $c_{P,2}(5) = 3.70$. Thus two future units from the sampled population will be located in the interval:

$$50.10 \pm (3.70)(1.31)$$

or 45.25 to 54.95, with a probability of 0.95.

The relative lengths of the three intervals obtained in the preceding examples are compared in Figure A.1. It is seen that for the given sample of 5, the confidence interval to contain the population mean is appreciably smaller than both the tolerance interval and the prediction interval. Also a tolerance interval to include at least 90 percent of the population with a probability of 0.95 is somewhat larger than a prediction interval to contain both of two future observations.

Inspection of the tabulations indicates that a confidence interval on the mean is always smaller than the other two intervals, but that the relative sizes of the tolerance and prediction intervals depend upon the proportion of the population to be contained in the prediction interval. Also, unlike the other two intervals, the length of a confidence interval approaches zero as the sample size increases (the interval converging to the point μ).

HOW TO SELECT THE RIGHT INTERVAL

The statistician's job is to develop correct procedures for answering relevant questions. The engineer must decide upon the relevant questions. Once the questions to be answered have been clearly stated, it should be easy to decide upon the correct intervals. The following comments are offered to serve as a guide to the engineer in this process.

The mean is the most commonly used single value to describe a population. For the normal distribution, the mean μ is one of the two parameters which uniquely defines the distribution. It is identical to the median (50 percent point) and mode (most common value) of the distribution. The population mean is therefore of great interest in characterizing product performance, and is often used as a standard by which competing processes are compared. Its use for such comparisons is especially appropriate when it is reasonable to assume that each of the competing processes has the same statistical variability (as measured by the process standard deviation) and, therefore, the differences between processes can be described completely by differences in their means. The assumption of equal standard deviations is frequently made.

Because of random fluctuations, a sample does not provide perfect information about the population mean μ. Thus, a confidence interval is established which contains the unknown value of μ with a specified degree of confidence.

If, instead of characterizing typical process performance, you are interested in estimating the range of variation of the underlying population or of the observations in a future sample, then a tolerance interval or a prediction interval is needed. Specifically, a tolerance interval is applicable if limits are needed that contain most of the sampled population, while a prediction interval would be used to obtain limits to contain all of a small number of future units from the population. Thus, an engineer who is concerned with the performance of a mass-produced item, such as a transistor or a lamp, would generally be interested in a

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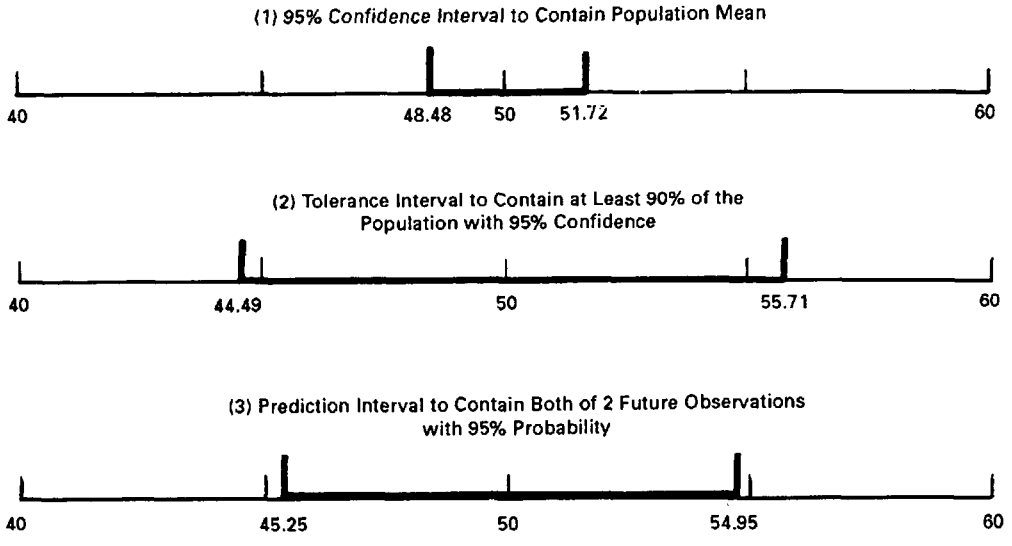

FIG. A.1 HOW THE LENGTHS OF THE STATISTICAL INTERVALS FOR THE EXAMPLE COMPARE

tolerance interval to enclose a high proportion of the sampled population.

In contrast, a prediction interval to contain all of k future observations may be thought of as the astronaut's interval. A typical astronaut, who has been assigned to a specific number of flights, is generally not very interested in what will happen on the average in the population of all space flights, of which his happen to be a random sample (confidence interval on the mean), or even what will happen in at least 99 percent of such flights (tolerance interval). His main concern is the worst that will happen in the one, three, or five flights in which he will be personally involved. Similarly, a turbine engineer who is bidding on an order of three units based upon his past experience on five units of the same type, would use a prediction interval to obtain specification limits to contain the performance parameter for all three units with a high probability. Prediction intervals are also required by the typical customer who purchases one or a small number of units of a given product and is concerned with predicting the performance of the particular units he has purchased (in contrast to the long-run performance of the process from which the sample has been selected).

WHERE TO GET MORE INFORMATION

Standard books on elementary engineering statistics, Reference 4, give prime space to the concept of confidence intervals and, in many cases, also discuss tolerance intervals, but make no mention of prediction intervals except in a regression context. Such intervals, however, are discussed in References 1, 2, 3, and 5. Further, Reference 3 provides a comprehensive comparison of statistical intervals for a normal population (including more detailed tabulations than are given here) and a discussion of methods for constructing the various intervals. This article also considers additional types of statistical intervals such as:

A prediction interval to contain a future sample mean,

A prediction interval to contain a future sample standard deviation,

A confidence interval for the population standard deviation,

A confidence interval for a population percentile.

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Finally, a new time-sharing computer program calculates a wide variety of statistical intervals, including confidence, tolerance, and prediction intervals, Reference 6.

THE EXAMPLE PROBLEM

The calculation of the three intervals are illustrated here by the following numerical example. Assume that readings obtained on a normally distributed performance parameter based on a random sample of five units are: 51.4, 49.5, 48.7, 49.3, and 51.6. From this information, the sample mean $\bar{y}$ and the sample standard deviation s are calculated by well-known expressions:

$$\bar{y} = \sum_{i=1}^{n} y_i/n = (51.4 + \ldots + 51.6)/5 = 50.10$$

$$s = \left[\frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n-1}\right]^{1/2}$$

$$= \left\{\frac{[(51.4 - 50.10)^2 + \ldots + (51.6 - 50.10)^2]}{(5-1)}\right\}^{1/2}$$

$$= 1.31,$$

where $y_1, \ldots, y_n$ are the values of n given observations.

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(1) Hahn, G. J., "Additional Factors for Calculating Prediction Intervals for Samples from a Normal Distribution," *Journal of the American Statistical Association,* 65, December, 1970.

(2) _____, "Factors for Calculating Two-Sided Prediction Intervals for Samples from a Normal Distribution." *Journal of the American Statistical Association,* 64, September, 1969.

(3) _____, "Statistical Intervals for a Normal Population," *Journal of Quality Technology,* Volume 2, Number 3, pages 115–125, July 1970; Volume 2, Number 4, pages 195–206, October 1970.

(4) Natrella, Mary Gibbons, *Experimental Statistics,* National Bureau of Standards Handbook 91, US Government Printing Office.

(5) Nelson, W. B., "Two-Sample Prediction," General Electric Company TIS Report 68-C-404, November 1968. (Available from Distribution Unit, PO Box 43, Building 5, Room 237, Schenectady, New York 12305).

(6) "Summary Statistics Package — ONE-SAM$***" Document 003401, General Electric Information Service Department, 7735 Old Georgetown Road, Bethesda, Maryland.

A.2 WEIGHTING METHOD

Whenever the value of a parameter is approximated by several different measuring methods, then weighted means of the systematic errors, random errors, and parameter uncertainties may be calculated.

Let $\bar{X}_i$ represent best estimates of a parameter by N measurement methods. Then $\bar{\bar{X}}$, the weighted mean of the measurements, can be given by

$$\bar{\bar{X}} = \sum_{i=1}^{N} W_i \bar{X}_i$$

where W_i are the required weighting factors.

A weighting principle which is statistically valid and is based on weighting by variances is applicable in this case [5]. This is true since the systematic uncertainty component of a parameter B is assumed to equal two times the standard deviation (square root of the variance) of the possible distribution of systematic uncertainty. The variance of this distribution is $(B/2)^2$. This is combined with $S_{\bar{X}}^2$, which is the variance of the average measurement. Therefore, U_{95} is a combination of variances:

$$U_{95} = t_{95}\left[\left(\frac{B}{2}\right)^2 + S_{\bar{X}}^2\right]^{1/2}$$

Therefore,

$$W_i = \frac{\left(\frac{1}{U_i}\right)^2}{\sum_{i=1}^{N}\left(\frac{1}{U_i}\right)^2} \qquad \text{(A.1)}$$

where U_i represents the uncertainties of $\bar{X}_i$ by the U_{95} model.

For two measurement methods with the means $\bar{X}_1$ and $\bar{X}_2$, Eq. (A.1) yields

$$W_1 = \frac{U_2^2}{U_1^2 + U_2^2}$$

and

$$W_2 = \frac{U_1^2}{U_1^2 + U_2^2} = 1 - W_1$$

Using these same weighting factors, the systematic and random uncertainties of the weighted mean are given by the root-sum-square relations

$$\bar{\bar{B}} = [W_1^2 B_1^2 + W_2^2 B_2^2]^{1/2}$$

and

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$$\overline{\overline{S}} = [W_1^2 S_1^2 + W_2^2 S_2^2]^{1/2}$$

where S_1 and S_2 are $S_{\overline{X}_1}$ and $S_{\overline{X}_2}$.

Similarly, the degrees of freedom of $\overline{\overline{U}}_{95}$ is given by the Welch-Satterthwaite formula:

$$\overline{\overline{\nu}} = \frac{\left[(W_1 S_1)^2 + (W_2 S_2)^2 + \left(W_1 \cdot \frac{B_1}{2}\right)^2 + \left(W_2 \cdot \frac{B_2}{2}\right)^2\right]^2}{\frac{(W_1 S_1)^4}{\nu_1} + \frac{(W_2 S_2)^4}{\nu_2} + \frac{\left(W_1 \cdot \frac{B_1}{2}\right)^2}{\nu_1} + \frac{\left(W_2 \cdot \frac{B_2}{2}\right)^4}{\nu_2}}$$

where S_1 and S_2 are $S_{\overline{X}_1}$ and $S_{\overline{X}_2}$.

These values are combined as U_{95} to obtain the weighted uncertainty of $\overline{\overline{X}}$ according to Eq. (4.5) as follows:

$$\overline{\overline{U}}_{95} = t_{95} [(\overline{\overline{B}}/2)^2 + \overline{\overline{S}}_{\overline{X}}^2]^{1/2}$$

where $t = 2$ if $\nu > 30$.

As usual, $\overline{\overline{S}}$, $\overline{\overline{B}}$, and $\overline{\overline{U}}_{95}$ should be reported.

A.3 OUTLIER TREATMENT

A.3.1 General. All measurement systems may produce spurious data points. These points may be caused by temporary or intermittent malfunctions of the measurement system. Errors of this type should not be included as part of the uncertainty of the measurement. Such points are considered to be meaningless as steady-state test data, and should be discarded. Figure A.1 shows a spurious data point called an outlier.

All data should be inspected for spurious data points as a continuing check on the measurement process. Identification criteria should be based on engineering analysis of instrumentation, thermodynamics, flow profiles, and past history with similar data. To ease the burden of scanning large masses of data, computerized routines are available to scan steady state data and flag suspected outliers. The suspected outliers should then be subjected to an engineering analysis.

The effect of outliers is to increase the standard deviation of the system. Tests are available to determine if a particular point from a sample is an outlier. In most of these tests, the probability for rejecting a good point is set at 5%. This means that the odds against rejecting a good point are 20 to 1 (or less). The odds could be increased by setting the probability of rejecting a good data point lower. However, this practice decreases the probability of rejecting bad data points. For small samples, bad data points are hard to identify.

Two tests are in common usage for determining whether or not spurious data are outliers. These are the Thompson τ Technique [18] and the Grubbs Method [19]. The Thompson τ Technique is excellent for rejecting outliers, but also may reject some good values. The Grubbs Method does not reject as many outliers but the number of good points rejected is smaller.

In this Supplement, the Modified Thompson τ Technique[1] described below is recommended for identifying possible outliers for further examination.

A.3.2 Thompson τ Technique (Modified). Consider a sample (X_i) of N measurements. The sample deviation (S_X) and the mean ($\overline{X}$) of the sample are calculated.

Suppose X_j, the jth observation, is the suspected outlier. Then, the absolute difference of X_j from the mean ($\overline{X}$) is calculated as

$$\delta = |X_j - \overline{X}|$$

Using Table A.2, a value of τ is obtained for the sample size (N) at the 5% significance level. This limits the probability of rejecting a good point to 5%. (The probability of not rejecting a bad data point is not fixed. It will vary as a function of sample size.)

The test for the outlier is to compare the difference (δ) with the product τS_X.

If δ is larger than or equal to τS_X, we say X_j is an outlier.

If δ is smaller than τS_X, we say X_j is not an outlier.

A.3.3 Outlier Examples. *Example 1:* There were 40 temperature probes installed in one stage of the turbine of a jet engine. The 40 probe readings were averaged and δ was calculated from the average for each probe.

26	79	58	24	1	−103	−121	−220
−11	−137	120	124	129	−38	25	−60
148	−52	−216	12	−56	89	8	−29
−107	20	9	−40	40	2	10	166
126	−72	179	41	127	−35	334	−555

−555 and 334 are suspected outliers.

[1] Thompson used a different equation for S. The Modified Thompson Technique uses the equation for S as defined in this Supplement.

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TABLE A.2
MODIFIED THOMPSON τ (AT THE 5% SIGNIFICANCE LEVEL)
(EXAMPLE B.3)

N	τ	N	τ
3	1.151	52	1.932
4	1.425	53	1.933
5	1.571	54	1.934
6	1.656	55	1.934
7	1.711	56	1.935
8	1.749	57	1.935
9	1.777	58	1.935
10	1.798	59	1.936
11	1.815	60	1.936
12	1.829	61	1.937
13	1.840	62	1.937
14	1.850	63	1.937
15	1.858	64	1.938
16	1.865	65	1.938
17	1.871	66	1.938
18	1.876	67	1.939
19	1.881	68	1.939
20	1.885	69	1.939
21	1.889	70	1.940
22	1.893	71	1.940
23	1.896	72	1.940
24	1.899	73	1.941
25	1.901	74	1.941
26	1.904	75	1.941
27	1.906	76	1.941
28	1.908	77	1.942
29	1.910	78	1.942
30	1.911	79	1.942
31	1.913	80	1.942
32	1.915	81	1.942
33	1.916	82	1.943
34	1.917	83	1.943
35	1.919	84	1.943
36	1.920	85	1.943
37	1.921	86	1.944
38	1.922	87	1.944
39	1.923	88	1.944
40	1.924	89	1.944
41	1.925	90	1.944
42	1.926	91	1.944
43	1.927	92	1.945
44	1.927	93	1.945
45	1.928	94	1.945
46	1.929	95	1.945
47	1.929	96	1.945
48	1.930	97	1.945
49	1.931	98	1.946
50	1.931	99	1.946
51	1.932	100	1.946
		∞	1.96

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To illustrate the calculations for determining whether −555 is an outlier, the following steps are taken.

Mean ($\overline{X}$) = 1.125
Sample standard deviation (S_x) = 140.8
Sample size (N) = 40

By the above equation for δ,

$$\delta = |-555 - 1.125| = 556.125$$

From Table A.2,

$$\tau S_X = 1.924 \times 140.8 = 270.9$$

Since $\delta > \tau S_X$, we conclude that −555 is an outlier according to the modified Thompson τ Technique.

Repeating the above procedure for 334,

$$\delta = |334 - 1.125| = 332.875$$

Since δ exceeds τS_X for the suspected point, 334, we conclude that 334 also is a probable outlier and also would be deleted.

This procedure would be continued until no more outliers are found. Whenever outliers are found, the validity of the data should be questioned.

Recalculate $\overline{X}$ and S_x when outliers are deleted and use these new values in reporting results. In the report on the test, a description of the use of the statistical data rejection should be given.

Example 2: In the following sample of 12 values from the distribution of a quality characteristic (data from [20]), the suspected outlier is 17.

3 4 5 6 7 7 8 9 9 10 11 17

Mean ($\overline{X}$) = 8
Sample standard deviation (S_x) = 3.717
Sample size (N) = 12

By the above equation for δ,

$$\delta = |17 - 8| = 9$$

From Table A.2,

$$\tau S_x = 1.829 \times 3.717 = 6.80$$

Since $\delta > \tau S_x$, we conclude that 17 is an outlier according to the modified Thompson τ Technique.

A.4 PARETO DIAGRAMS

It is often useful to display the relative sizes of the components of a whole with a bar chart. One particular type of bar chart is called a Pareto diagram after Vilfredo Pareto, an Italian economist, who used this type of diagram in his studies of the unequal distribution of wealth. Most of the uses today, with extensive activity in the area of quality control, are attributed to Joseph Juran who defined the general principle known as the "Pareto Principle" — the "Vital Few, Trivial Many." Mathematics were developed which described the distribution, but for the purposes illustrated here, the diagram can be defined as a bar chart with the bars arranged in descending order of size.

To apply this to a measurement uncertainty example, the first step is to define the individual systematic and random uncertainties of the mean in terms of their relative individual percentages of the total uncertainty, U_{95}.

To derive the percentages for U_{95}, the calculations must be made in terms of U_{95}^2. The reason for this is that

$$U_{95} = 2\sqrt{\left(\frac{B}{2}\right)^2 + S_R^2}$$

Since the squares of the random and systematic uncertainties are additive under the square root, the total uncertainty must be squared before making the calculation.

First, for the systematic uncertainties, B_i, define

$$\frac{4\left(\frac{B_i\theta_i}{2}\right)^2}{4\left(\frac{B}{2}\right)^2}$$

where

$$B^2 = \sum(B_i\theta_i)^2$$

And that term is the following fraction of total uncertainty squared, U_{95}^2

$$\frac{4\left(\frac{B}{2}\right)^2}{U_{95}^2} = \text{the systematic component of } U_{95}^2$$

where

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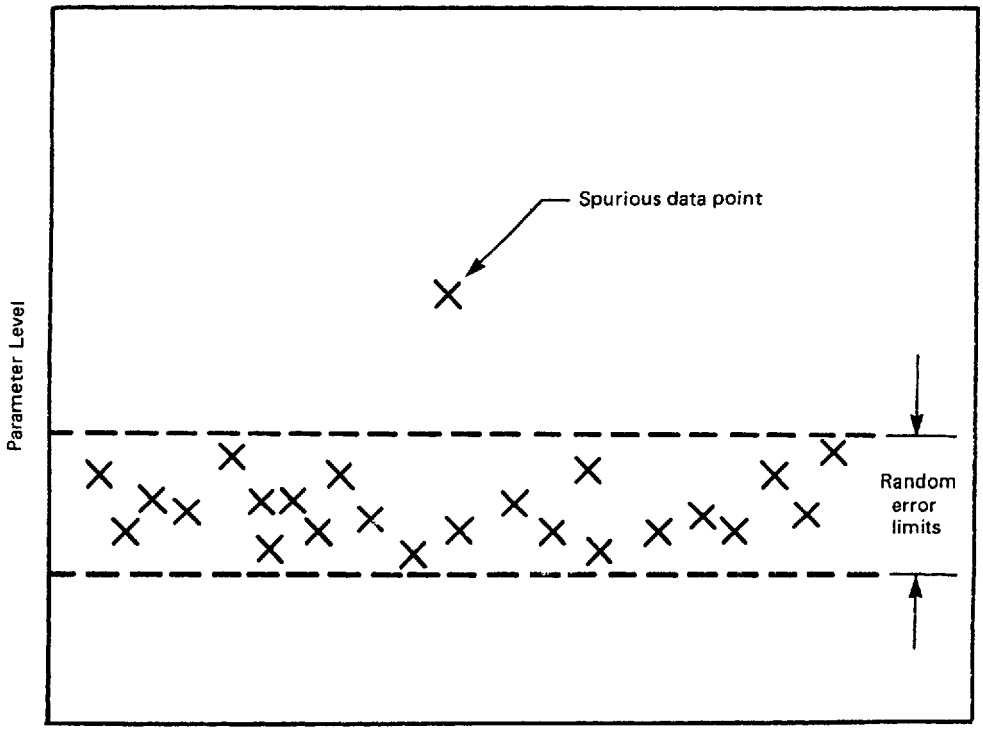

FIG. A.2 OUTLIER OUTSIDE THE RANGE OF ACCEPTABLE DATA

$$U_{95}^2 = 4\left(\frac{B}{2}\right)^2 + 4\,S_R^2$$

and where

$$4\,S_R^2 = 4\sum_{i=1}^{N}(S_{\bar{X}_i}\theta_i)^2$$

$4\,S_R^2$ = the random component of U_{95}^2

The percentage of U_{95}^2 for each individual systematic uncertainty is

$$\frac{4\left(\frac{B_i\theta_i}{2}\right)^2}{4\left(\frac{B}{2}\right)^2} \times \frac{4\left(\frac{B}{2}\right)^2}{U_{95}^2} \times 100 = \frac{4\left(\frac{B_i\theta_i}{2}\right)^2}{U_{95}^2} \times 100$$

For the random uncertainty portion of total uncertainty, the fraction is defined as

$$\frac{4\,(S_{\bar{X}_i}\theta_i)^2}{4\,S_R^2}$$

where

$$4\,S_R^2 = 4\sum(S_{\bar{X}_i}\theta_i)^2$$

And that term is the following fraction of total uncertainty squared, U_{95}^2

$$\frac{4\,S_R^2}{U_{95}^2}$$

The percentage of U_{95}^2 is

$$\frac{4\,(S_{\bar{X}_i}\theta_i)^2}{4\,S_R^2} \times \frac{4\,S_R^2}{U_{95}^2} \times 100 = \frac{4\,(S_{\bar{X}_i}\theta_i)^2}{U_{95}^2} \times 100$$

Proof that this sum is 1, or 100%, for a case of three individual systematic and random uncertainties is shown below:

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B_1, B_2, B_3 and $S_{\bar{X}_1}$, $S_{\bar{X}_2}$, $S_{\bar{X}_3}$ and $\theta_1 = \theta_2 = \theta_3 = 1$

$$U_{95}^2 = 4\left(\frac{B}{2}\right)^2 + 4\,S_R^2$$

$$B_1\%\text{ of }U_{95}^2 = \frac{4\left(\frac{B_1}{2}\right)^2}{U_{95}^2} \times 100 = \frac{4\left(\frac{B_1}{2}\right)^2 \times 100}{4\left(\frac{B}{2}\right)^2 + 4\,S_R^2}$$

$$S_{x1}\%\text{ of }U_{95}^2 = \frac{4\,(S_{x1})^2\,100}{4\left(\frac{B}{2}\right)^2 + 4\,S_R^2}$$

For the sum:

$$\text{Sum} = B_1\% + B_2\% + B_3\% + S_{x1}\% + S_{x2}\% + S_{x3}\%$$

$$\text{Sum} = \frac{4\left[\left(\frac{B_1}{2}\right)^2 + \left(\frac{B_2}{2}\right)^2 + \left(\frac{B_3}{2}\right)^2\right] \times 100}{4\left(\frac{B}{2}\right)^2 + 4\,S^2} + \frac{4\left[S_{\bar{X}_1}^2 + S_{\bar{X}_2}^2 + S_{\bar{X}_3}^2\right] \times 100}{4\left(\frac{B}{2}\right)^2 + 4\,S_R^2} = \frac{4\left[\left(\frac{B}{2}\right)^2 + 4\,S_R^2\right]}{4\left(\frac{B}{2}\right)^2 + 4\,S_R^2} = 100\%$$

Using the compressor performance example in para. 10.4 for illustration:

Parameter	Absolute Random Uncertainty of the Mean	Absolute Systematic Uncertainty	Absolute Sensitivity
P_1	0.03 psia	0.02 psia	−0.0409
P_2	0.17 psia	0.17 psia	+0.00629
T_1	0.6°F	0.8°F	+0.00367
T_2	0.7°F	1.6°F	−0.00203

Step 1: For $B_{P_1}\%$ of U_{95}

$$B = [(\theta_{P_1} \times B_{P_1})^2 + (\theta_{P_2} \times B_{P_2})^2 + (\theta_{T_1} \times B_{T_1})^2 + (\theta_{T_2} \times B_{T_2})^2 + 2\theta_{T_1}\theta_{T_2}\,B_{T_1T_2}]$$

where

$$B_{T_1T_2} = [(0.5)\,(0.05) + (0.5)\,(0.5) + (0.07)\,(0.07) + (0.4)\,(0.4)] = 0.665\ (\text{F})^2$$

$B_{T_1T_2}$ is the covariance of error sources common to T_1 and T_2

$$B = [(-0.0409 \times 0.02)^2 + (0.00629 \times 0.17)^2 + (0.00367 \times 0.8)^2 + (-0.00203 \times 1.6)^2 + 2(0.00367)\,(-0.00203)\,(0.665)]^{1/2} = +0.003$$

$$S_x = [(-0.0409 \times 0.03)^2 + (0.00629 \times 0.17)^2 + (0.00367 \times 0.6)^2 + (-0.00203 \times 0.7)^2]^{1/2} = 0.003$$

Therefore

$$U_{95}^2 = 4\left(\frac{B}{2}\right)^2 + 4\,S_{\bar{X}}^2 = 0.00005$$

$$B_{P_1}\%\text{ of }U_{95} = \frac{4\left(\frac{B_{P_1}\theta_{P_1}}{2}\right)^2}{U_{95}^2} \times 100 = \frac{4\left(\frac{0.02 \times -0.0409}{2}\right)^2}{0.0000495} \times 100 = 1.35\%$$

Similarly,

$$B_{P_2} = 2.31\%,\ B_{T_1} = 17.41\%,\ B_{T_2} = 21.31\%$$

and

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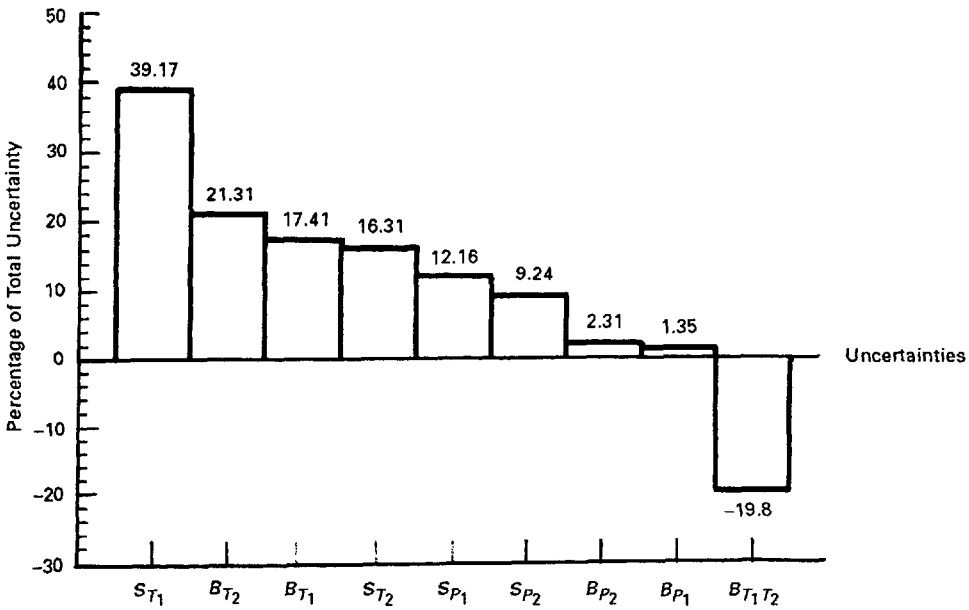

FIG. A.3 PARETO CHART FOR RANDOM AND SYSTEMATIC UNCERTAINTIES (EXAMPLE 10.4)

$B_{T_1T_2}$% of U_{95}

$$= 4\left[\frac{2(0.00367)(-0.00203)(1)(0.665/4)}{0.00005}\right] \times 100$$

$$= -19.8\%$$

Step 2: For $S_{\bar{P}_1}$% of U_{95}

$$S_{\bar{P}_1}\% \text{ of } U_{95} = \frac{(4\, S_{\bar{P}_1}\, \theta_{P_1})^2}{U_{95}{}^2} \times 100$$

$$= \frac{4\,(0.03 \times -0.0409)^2}{0.0000495} \times 100$$

$$= 12.16\%$$

Similarly,

$$S_{\bar{P}_2} = 9.24\%,\ S_{\bar{T}_1} = 39.17\%,\ S_{\bar{T}_2} = 16.31\%$$

Figure A.3 illustrates the Pareto charts for the percentages calculated for U_{95}.

However, since the systematic uncertainties for T_1 and T_2 are correlated, the relative percentages for B_{T_1}, B_{T_2}, and $B_{T_1T_2}$ should be combined as shown in Fig. A.4.

As can be seen from this diagram, S_{T_1} is the largest contributor to the total uncertainty, U_{95}, and B_{P_1} the smallest. The ideal end result of an analysis such as this would be to take corrective action, if possible, to reduce the contribution of major factors in the final uncertainty through changes in methods, instrumentation, or both. Although this application of Pareto diagrams has been used to determine the relative contributions to total uncertainty of systematic and random uncertainties, the method can be applied just as easily to the individual estimates of the elemental errors that contribute to systematic and random uncertainties.

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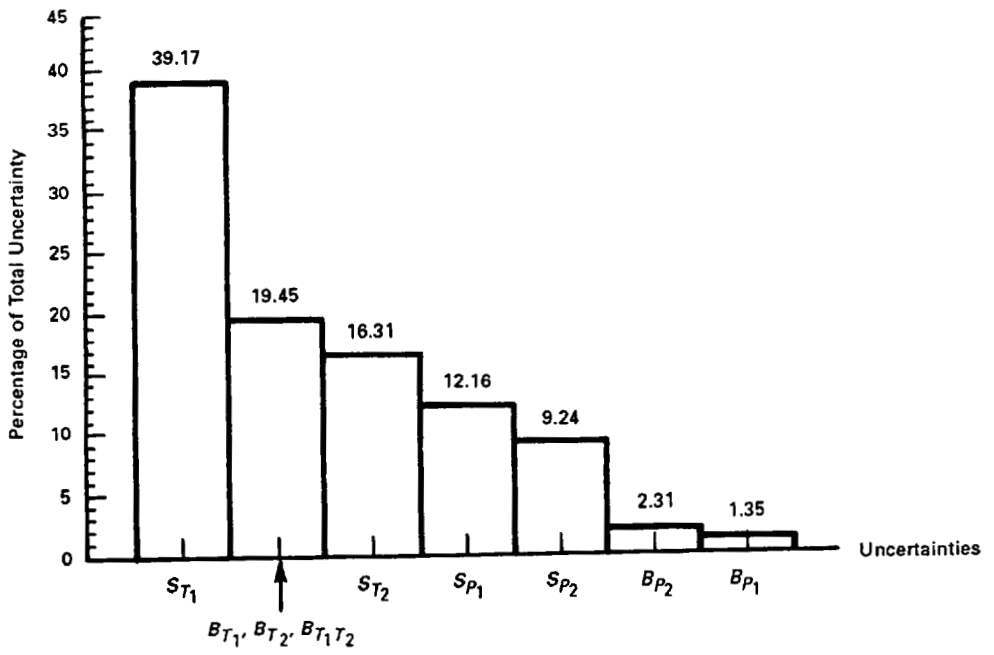

FIG. A.4 PARETO CHART FOR RANDOM AND SOME COMBINED SYSTEMATIC UNCERTAINTIES (EXAMPLE 10.4)

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APPENDIX B — UNCERTAINTY ANALYSIS MODELS

(This Appendix is not a part of ASME PTC 19.1-1998 and is included for information purposes only.)

B.1 ISO UNCERTAINTY ANALYSIS MODEL

Appendix B is adapted from [21]. The uncertainty model given in the ISO *Guide to the Expression of Uncertainty in Measurement* (1993) [1] is presented in this paragraph. The uncertainty model requires estimates of the uncertainties for each of the elemental error sources for each parameter in the data reduction equation. These estimates are combined through use of the ISO model to calculate the band about the experimental result where the true result is thought to lie with C% confidence. The estimates of the elemental errors fall into two categories, systematic uncertainties, $(b_i)_k$, for the fixed errors and sample standard deviations of the mean, S_i, for the random errors, where i represents each parameter.

The systematic uncertainties are related to those fixed errors that remain after all calibration corrections are made. Systematic uncertainties can be estimated through manufacturer information, calibrations, and, in most cases, through sound engineering judgment. There will usually be a set, K_i, of elemental systematic uncertainties for each parameter, i. As discussed in paras. 6.2 and 6.3, each elemental estimate, $(b_i)_k$, is taken to be the prediction of the 95% limits on a particular distribution of possible errors for that particular error source. Typically, these error distributions are assumed Gaussian (normally distributed) or rectangular (uniformly distributed). The ISO model uses the standard deviation of the assumed distribution, not the 95% estimate, to calculate the uncertainty in the result. Obtaining the standard deviation from the 95% estimate is simply a matter of dividing the estimate by the appropriate distribution factor, D [i.e., 2.0 for Gaussian, 1.65 for rectangular (1.73)(0.95)]. The systematic uncertainty for parameter i at the standard deviation level, $(S_B)_i$, is determined from the estimates for the K_i elemental error sources for that parameter as

$$[(S_B)_i]^2 = \sum_{k=1}^{K_i} [(b_i)_k/D_k]^2 \qquad \text{(B.1)}$$

Note that D_k can be different for each elemental error source.

The estimate of the random error for a parameter is the sample standard deviation of the mean, or the estimate of the error associated with the repeatability of a particular measurement. The sample standard deviation of the mean for each parameter is determined from N_i readings as

$$S_i = \left\{\left[\frac{1}{N_i(N_i-1)}\right]\sum_{k=1}^{N_i}[(X_i)_k - \overline{X}_i]^2\right\}^{1/2} \qquad \text{(B.2)}$$

where

$$\overline{X}_i = \frac{\sum_{k=1}^{N_i}(X_i)_k}{N_i} \qquad \text{(B.3)}$$

For the case of single readings ($N_i = 1$), previous information must be used to calculate S_i [4].

Consider an experimental result that is determined from j measured variables as

$$R = R(\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_j) \qquad \text{(B.4)}$$

The ISO Guide defines the combined standard uncertainty as

$$u_R^2 = \sum_{i=1}^{j}[\theta_i(S_B)_i]^2 + 2\sum_{i=1}^{j-1}\sum_{k=i+1}^{j}\theta_i\theta_k(S_B)_{ik} + \sum_{i=1}^{j}(\theta_i S_i)^2 \qquad \text{(B.5)}$$

where

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$$\theta_i = \frac{\partial R}{\partial \bar{X}_i} \tag{B.6}$$

The first two terms on the right side of Eq. (B.5) represent the systematic uncertainty of the result at the standard deviation level. The third term is the standard deviation of the result, or the random uncertainty at the standard deviation level. The covariance of the random errors is assumed to be zero. The covariance of the systematic errors, or the correlated systematic uncertainty, $(S_B)_{ik}$, is determined by summing the products of the elemental systematic uncertainties at the standard deviation level for parameters i and k that arise from the same source and are therefore perfectly correlated [9] (see para. 8.1).

In order to obtain the overall uncertainty in the result, U_R, at a specified confidence level, the ISO Guide recommends that the combined standard uncertainty be multiplied by a coverage factor. The coverage factor is the value from the t distribution for the required confidence level corresponding to the effective degrees of freedom in the result, ν_R. The values are given in Table B.1.

To find ν_R, the Welch-Satterthwaite formula is used:

$$\nu_R = \frac{\left[\sum_{i=1}^{j} \{[\theta_i (S_B)_i]^2 + (\theta_i S_i)^2\}\right]^2}{\sum_{i=1}^{j} \left[\frac{(\theta_i S_i)^4}{\nu_{S_i}} + \sum_{k=1}^{K_i} \frac{(\theta_i [b_i)_k / D_k])^4}{\nu_{(b_i)_k}}\right]} \tag{B.7}$$

where ν_{S_i} is either

$$\nu_{S_i} = N_i - 1 \tag{B.8}$$

or the degrees of freedom of the previous information if S_i is estimated [4]. The degrees of freedom of the systematic uncertainties $\nu_{(b_i)_k}$ may be known from previous information or estimated. The ISO Guide recommends the approximation

$$\nu_{(b_i)_k} = \frac{1}{2}\left(\frac{\Delta(b_i)_k}{(b_i)_k}\right)^{-2} \tag{B.9}$$

where the quantity in parentheses is an estimate of the relative variability of estimates of $(b_i)_k$. For instance, if one thought that the estimate of $(b_i)_k$ was reliable to within ±25%, then

TABLE B.1
STUDENT'S t DISTRIBUTION [8]

ν \ C	0.900	0.950	0.990	0.995	0.999
1	6.314	12.706	63.657	127.321	636.619
2	2.920	4.303	9.925	14.089	31.598
3	2.353	3.182	5.841	7.453	12.924
4	2.132	2.776	4.604	5.598	8.610
5	2.015	2.571	4.032	4.773	6.869
6	1.943	2.447	3.707	4.317	5.959
7	1.895	2.365	3.499	4.029	5.408
8	1.860	2.306	3.355	3.833	5.041
9	1.833	2.262	3.250	3.690	4.781
10	1.812	2.228	3.169	3.581	4.587
11	1.796	2.201	3.106	3.497	4.437
12	1.782	2.179	3.055	3.428	4.318
13	1.771	2.160	3.012	3.372	4.221
14	1.761	2.145	2.977	3.326	4.140
15	1.753	2.131	2.947	3.286	4.073
16	1.746	2.120	2.921	3.252	4.015
17	1.740	2.110	2.898	3.223	3.965
18	1.734	2.101	2.878	3.197	3.922
19	1.729	2.093	2.861	3.174	3.883
20	1.725	2.086	2.845	3.153	3.850
21	1.721	2.080	2.831	3.135	3.819
22	1.717	2.074	2.819	3.119	3.792
23	1.714	2.069	2.807	3.104	3.768
24	1.711	2.064	2.797	3.090	3.745
25	1.708	2.060	2.787	3.078	3.725
26	1.706	2.056	2.779	3.067	3.707
27	1.703	2.052	2.771	3.057	3.690
28	1.701	2.048	2.763	3.047	3.674
29	1.699	2.045	2.756	3.038	3.659
30	1.697	2.042	2.750	3.030	3.646
40	1.684	2.021	2.704	2.971	3.551
60	1.671	2.000	2.660	2.915	3.460
120	1.658	1.980	2.617	2.860	3.373
∞	1.645	1.960	2.576	2.807	3.291

GENERAL NOTE: Given are the values of t for a confidence level C and number of degrees of freedom ν.

$$\nu_{(b_i)_k} = \frac{1}{2}(0.25)^{-2} = 8 \tag{B.10}$$

With ν_R known, the proper t value is obtained from Table B.1 for C% confidence and multiplied by u_R from Eq. (B.5) to obtain the overall uncertainty in the result, U_R, at a C% confidence level

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$$U_{R_C}^2 = t_C^2 \left\{ \sum_{i=1}^{j} [\theta_i (S_B)_i]^2 + 2 \sum_{i=1}^{j-1} \sum_{k=i+1}^{j} \theta_i \theta_k (S_B)_{ik} + \sum_{i=1}^{j} (\theta_i S_i)^2 \right\} \quad \text{(B.11)}$$

Separating the systematic and random terms in Eq. (B.11) and taking the square root yields

$$B_{R_C} = t_C \left\{ \sum_{i=1}^{j} [\theta_i (S_B)_i]^2 + 2 \sum_{i=1}^{j-1} \sum_{k=i+1}^{j} \theta_i \theta_k (S_B)_{ik} \right\}^{1/2} \quad \text{(B.12)}$$

and

$$(t_C S_R) = t_C \left[\sum_{i=1}^{j} (\theta_i S_i)^2 \right]^{1/2} \quad \text{(B.13)}$$

where B_{R_C} represents the systematic uncertainty of the result and $t_C S_R$ represents the random uncertainty of the result each at C% confidence level.

B.2 LARGE SAMPLE UNCERTAINTY ANALYSIS APPROXIMATION

The method described in para. B.1 above is the strict ISO method. In the previous version of this standard [2], the recommendation was made that the t value for 95% confidence with 30 or more degrees of freedom could be taken as 2. Therefore, for large degrees of freedom in the result ($\nu_R \geq 30$), Eq. (B.11) becomes (assuming a Gaussian distribution for all systematic uncertainties)

$$U_{R_{95}}^2 = (2)^2 \left\{ \sum_{i=1}^{j} \left(\theta_i \frac{B_i}{2} \right)^2 + 2 \sum_{i=1}^{j-1} \sum_{k=i+1}^{j} \theta_i \theta_k \frac{B_{ik}}{4} + \sum_{i=1}^{j} (\theta_i S_i)^2 \right\} \quad \text{(B.14)}$$

The large sample systematic uncertainty of the result at 95% confidence is then

$$B_{R_{95}} = \left[\sum_{i=1}^{j} (\theta_i B_i)^2 + 2 \sum_{i=1}^{j-1} \sum_{k=i+1}^{j} \theta_i \theta_k B_{ik} \right]^{1/2} \quad \text{(B.15)}$$

which is equivalent to Eq. (8.2) and the 95% random uncertainty estimate is

$$(2S_R) = \left[\sum_{i=1}^{j} [\theta_i (2S_i)]^2 \right]^{1/2} \quad \text{(B.16)}$$

which is Eq. (7.7) multiplied by 2. By squaring Eqs. (B.15) and (B.16) and substituting into Eq. (B.14), the large sample uncertainty expression given in Eq. (7.14) is obtained (including the correlated systematic uncertainty terms)

$$U_{R_{95}} = 2 \left[\left(\frac{B_{R_{95}}}{2} \right)^2 + (S_R)^2 \right]^{1/2} \quad \text{(B.17)}$$

It has been shown [22–24] that for most engineering applications, when the degrees of freedom for the result from Eq. (B.7) is 9 or greater, t_{95} can still be taken as 2 to a good approximation (93% to 95% coverage) and the large sample approximation given by Eq. (B.17) can be used.

B.3 EXAMPLE PROBLEM

To demonstrate the ISO Uncertainty Analysis Model given in para. B.1, an example is presented of the experimental determination of the loss coefficient for water flow through a globe valve, C_{gv}, as shown in Fig. B.1. The loss coefficient is determined from the expression

$$C_{gv} = \left(\frac{\pi^2}{8} \right) \left[\frac{g t^2 d^4 \rho^2}{(12)^5 m^2} \right] [(h_1 - h_2) - (h_3 - h_4)] \quad \text{(B.18)}$$

where g is the acceleration of gravity, t is the time required to fill a container with the water flow during the duration of the experiment, d is the pipe inner diameter, ρ is the density of the water, m is the mass of the water in the container at the end of the experiment, h_1 is the flow head entering the globe valve, and h_2 is the flow head exiting the globe valve. Also included in the above equation are h_3 and h_4, the entering and exiting head measurements across a section of pipe equivalent in length to the pipe that penetrates the globe valve fittings. This head loss was deducted from the drop across the valve since it was found in preliminary investigations to have a significant effect on the calculation of the loss coefficient.

The flow heads were measured by extending tubing vertically from the taps against a surface that had been scaled for measuring purposes, with all four tubes being open to ambient pressure. The

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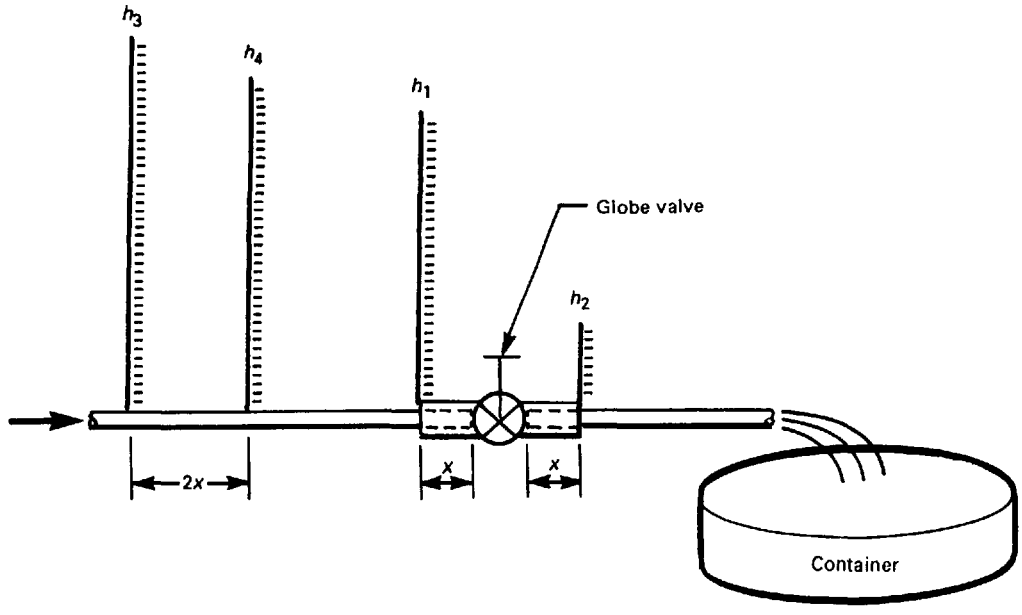

FIG. B.1 SCHEMATIC OF EXPERIMENT APPARATUS (EXAMPLE B.3)

valve was fully open, allowing water to flow through the apparatus and into the container. The duration of the experiment was dependent upon how long it took to fill the container and was measured with a handheld stopwatch. Once the container was filled, the mass of water collected was measured.

In pretest studies, it was found that the uncertainty in the pipe inner diameter was a major contributor to the uncertainty in C_{gv}. Therefore, it was necessary to determine the mean diameter other than by direct measurement. The length and volume of a section of pipe from the same lot as that used in the experiment were used to determine the mean diameter. This auxiliary test was repeated nine times.

During the experiment, six multiple readings of the head measurements were made in order to obtain the mean and standard deviation for each. The random uncertainty estimate in the time measurement $2S_t$, attributed to the reaction of the person timing the event, was estimated using good judgment while the random uncertainties in the diameter and density were taken to equal zero. The random uncertainty of the mass, $2S_m$, was estimated based on the ability to read the scale. The systematic uncertainty estimates for the head measurements were taken to result from the accuracy of the scale used for the measurements and the variability in the placement of the taps on the test apparatus. The systematic uncertainty in the time measurement was taken as the accuracy information provided by the manufacturer of the stopwatch. For the diameter, the estimate of the standard deviation of the mean provided by the repeated diameter measurements was multiplied by a factor of 2 and "fossilized" into a systematic uncertainty with 8 degrees of freedom (nine readings). The systematic uncertainty in the density was estimated using good judgment while that of the mass measurement was provided from manufacturer's information concerning the accuracy of the weight scale. The mass systematic uncertainty was taken to be uniformly distributed while the others were taken to be Gaussian. The inputs for the variable measurements and their respective elemental uncertainty estimates are listed in Table B.2 along with the degrees of freedom of each estimate. A degrees of freedom of 50 was arbitrarily assigned to each uncertainty that was estimated.

With the random and systematic uncertainties for each variable determined, the uncertainty analysis was performed using Eq. (B.11) for a 95% confidence level. For this example, the systematic uncertainties attributed to the head measurements are correlated because the same measuring scale was used to graduate each water column. The results for the uncertainty analysis are given in Fig. B.2. The plot

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TABLE B.2
DETAILED UNCERTAINTY ANALYSIS INPUT (EXAMPLE B.3)

Symbol	Values	95% Systematic Uncertainty Estimate, B_i	ν_{B_i}	Standard Deviation Estimate, S_i	ν_{S_i}
(1) t	72.72 (sec)	0.01 (sec)	50	0.1 (sec)	50
(2) d	0.547 (in.)	0.00308 (in.)	8	0 (in.)	NA
(3) ρ	62.3 (lbm/ft^3)	0.05 (lbm/ft^3)	50	0 (lbm/ft^3)	NA
(4) m	50.0 (lbm)	0.5 (lbm)	50	0.2 (lbm)	50
(5) h_1	125.3 (in.)	0.375 (in.)	50	0.125 (in.)	5
(6) h_2	11.2 (in.)	0.375 (in.)	50	0.1 (in.)	5
(7) h_3	137.2 (in.)	0.375 (in.)	50	0.1 (in.)	5
(8) h_4	130.5 (in.)	0.375 (in.)	50	0.15 (in.)	5

NA = not applicable

of the relative contributions shows the contributions of the terms shown below:

$$\theta_i^2 S_i^2;\ \theta_i^2 (S_B)_i^2;\ 2\theta_i \theta_k (S_B)_{ik}$$

Each term is normalized by the largest of these terms. The plot of the normalized contributions for this example is shown in Fig. B.2 along with its legend and the uncertainty analysis results.

Note that the degrees of freedom of the result is 52 even though some of the variable degrees of freedom are small. In this case, t_{95} is equal to 2.0.

B.4 COMPARISON OF UNCERTAINTY MODELS

The recommended uncertainty interval equation [Eq. (B.17)] differs from the uncertainty interval equations presented in the earlier version of this standard [2]. Previously, two equations were suggested, U_{ADD} corresponding to 99% coverage and U_{RSS} corresponding to 95% coverage. These equations were

$$U_{ADD} = B + t_{95} S_{\bar{X}} \tag{B.19}$$

and

$$U_{RSS} = \sqrt{(B)^2 + (t_{95} S_{\bar{X}})^2} \tag{B.20}$$

To simplify the uncertainty analysis process, this document has standardized to one equation corresponding to 95% confidence with large degrees of freedom. Details are provided in this Appendix to construct intervals with alternate confidence levels and with small degrees of freedom.

Although the recommended uncertainty equation appears different in form, it is important to note that the U_{RSS} method and the new recommended U_{95} equation converge to the same answer for the case with large degrees of freedom. The interval size is identical.

Monte Carlo simulation comparisons of the U_{ADD} and U_{RSS} uncertainty models against the ISO model were presented in [22]. This simulation work indicated more consistent coverage for the ISO model as compared to the U_{ADD} and U_{RSS} techniques for a wide range of assumptions for the degrees of freedom, different distribution forms for the systematic errors, and varying magnitudes of the systematic and random components. It was also shown that the U_{95} model in Eq. (B.17) had a reasonably good comparison with the ISO model for 95% confidence. For this reason, U_{95} is preferred over both U_{ADD} and U_{RSS}.

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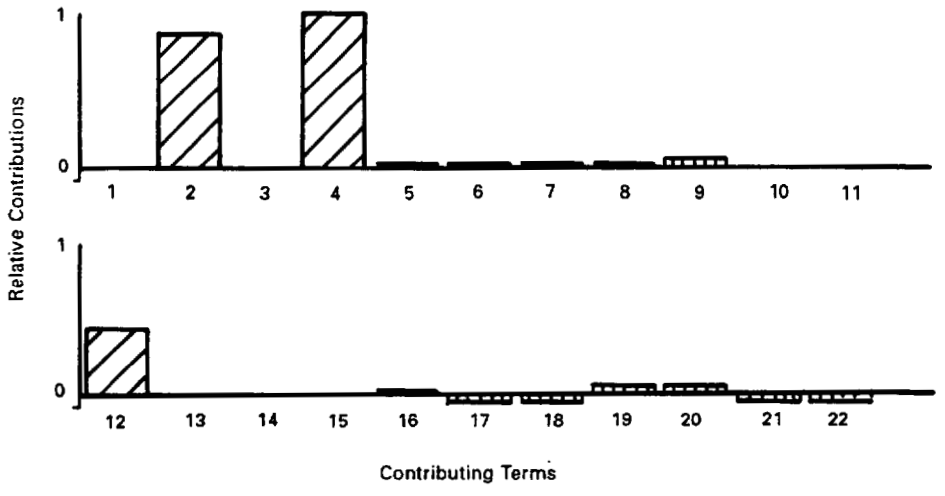
Graph Output

Normalized Contributions to Uncertainty in Result



Graph Legend

Identifying Labels and Normalized Contribution for Each Term

1 = Sys. Term (t) = 0.0%	9 = Ran. Term (t) = 5.2%	17 = Cor. Term (5,6) = −4.2%
2 = Sys. Term (d) = 86.5%	10 = Ran. Term (d) = 0.0%	18 = Cor. Term (5,7) = −4.2%
3 = Sys. Term (ρ) = 0.4%	11 = Ran. Term (ρ) = 0.0%	19 = Cor. Term (5,8) = 4.2%
4 = Sys. Term (m) = 100.0%	12 = Ran. Term (m) = 43.6%	20 = Cor. Term (6,7) = 4.2%
5 = Sys. Term (h_1) = 2.1%	13 = Ran. Term (h_1) = 0.9%	21 = Cor. Term (6,8) = −4.2%
6 = Sys. Term (h_2) = 2.1%	14 = Ran. Term (h_2) = 0.6%	22 = Cor. Term (7,8) = −4.2%
7 = Sys. Term (h_3) = 2.1%	15 = Ran. Term (h_3) = 0.6%	
8 = Sys. Term (h_4) = 2.1%	16 = Ran. Term (h_4) = 1.3%	

Uncertainty Analysis Results

C_{gv} = 12.6	$U_{C_{gv}}$ = 0.472 (3.75%)
$\nu_{C_{gv}}$ = 52	$B_{C_{gv}}$ = 0.417 (3.31%)
	$t_{95}S_{C_{gv}}$ = 0.220 (1.75%)

FIG. B.2 DETAILED UNCERTAINTY ANALYSIS RESULTS (EXAMPLE B.3)

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APPENDIX C — PROPAGATION OF UNCERTAINTY THROUGH TAYLOR SERIES

(This Appendix is not a part of ASME PTC 19.1-1998 and is included for information purposes only.)

C.1 INTRODUCTION

Experimental results are not always directly measured. It is quite common for an experimental result, $r(X_1, X_2, \ldots, X_n)$, to be defined as a function of certain variables, $X_1, X_2, \ldots, X_n$, that are directly measured. The aim of this Appendix is to provide a method by which the variance of an experimental result that is not directly measured, $r(X_1, X_2, \ldots, X_n)$, can be expressed in terms of the variances and covariances of its arguments, $X_1, X_2, \ldots, X_n$, which are directly measured.

The approach will be to relate the deviations in $r(X_1, X_2, \ldots, X_n)$ to deviations in $(X_1, X_2, \ldots, X_n)$ by means of a first order approximation to the Taylor series expansion for $r(x_1, x_2, \ldots, x_n)$ in the neighborhood of the point $(\mu_{X1}, \mu_{X2}, \ldots, \mu_{Xn})$, where μ_{Xi} is the true value of the measured variable X_i. In order to facilitate this project, the function $r(x_1, x_2, \ldots, x_n)$ will be assumed to be continuous with continuous partial derivatives in the neighborhood of the point $(\mu_{X1}, \mu_{X2}, \ldots, \mu_{Xn})$.

C.2 DEFINITIONS

The primary goal of Appendix C is to present an expression for the variance in r in terms of the variances and covariances of $X_1, X_2, \ldots, X_n$. The definitions below for "mean," "variance," and "covariance" will be used throughout this Appendix.

The expected value of a function $f(X)$ of a random variable X is given by

$$E[f(X)] = \int_{-\infty}^{\infty} f(x)\, p(x)\, dx$$

where $p(x)$ is the probability density function for X.

The mean (or expected value) μ_X of the random variable X presents the special case where $f(X) = X$, i.e.,

$$E[X] = \int_{-\infty}^{\infty} x\, p(x)\, dx = \mu_X$$

The variance σ_X^2 of the random variable X presents the special case where $f(X) = (X - \mu_X)^2$, i.e.,

$$E[(X - \mu_X)^2] = \int_{-\infty}^{\infty} (x - \mu_X)^2\, p(x)\, dx = \sigma_X^2$$

The expected value of a function $f(X_1, X_2, \ldots, X_n)$ of the random variables $X_1, X_2, \ldots, X_n$ is given by

$$E[f(X_1, X_2, \ldots, X_n)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots \int_{-\infty}^{\infty} f(x_1, x_2, \ldots, x_n)\, p(x_1, x_2, \ldots, x_n)\, dx_1\, dx_2 \ldots dx_n$$

where $p(x_1, x_2, \ldots, x_n)$ is the joint probability density function for $X_1, X_2, \ldots, X_n$.

The covariance σ_{X1X2} of the random variables X_1, X_2 presents the special case where $f(X_1, X_2) = (X_1 - \mu_{X1})(X_2 - \mu_{X2})$, i.e.,

$$E[f(X_1, X_2)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x_1 - \mu_{X1})(x_2 - \mu_{X2})\, p(x_1, x_2)\, dx_1\, dx_2 = \sigma_{X1X2}$$

C.3 PRELIMINARY CONSIDERATIONS

If any random variable, Y, can be expressed as a linear combination of random variables, X_i, then the mean and variance of Y can be expressed in

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terms of the means, variances, and covariances of the variables X_i.

Suppose

$$Y = a_0 + a_1 X_1 + a_2 X_2 + \ldots + a_n X_n$$

Then

$$E[Y] = E[a_0 + a_1 X_1 + a_2 X_2 + \ldots + a_n X_n]$$

or, using the definition of the mean value of a random variable,

$$\mu_Y = a_0 + a_1 \mu_{X1} + a_2 \mu_{X2} + \ldots + a_n \mu_{Xn}$$

The variance of Y will be given as follows:

$$\begin{aligned}\sigma_Y^2 &= E[(Y - \mu_Y)^2] \\ &= E[\{a_1 (X_1 - \mu_{X1}) + a_2 (X_2 - \mu_{X2}) \\ &\quad + \ldots + a_n (X_n - \mu_{Xn})\}^2]\end{aligned}$$

For example, if $n = 3$, then

$$\begin{aligned}\sigma_Y^2 &= E[\{a_1 (X_1 - \mu_{X1}) \\ &\quad + a_2 (X_2 - \mu_{X2}) + a_3 (X_3 - \mu_{X3})\}^2] \\ \sigma_Y^2 &= E[a_1^2 (X_1 - \mu_{X1})^2 + a_2^2 (X_2 - \mu_{X2})^2 \\ &\quad + a_3^2 (X_3 - \mu_{X3})^2 \\ &\quad + 2a_1 a_2 (X_1 - \mu_{X1})(X_2 - \mu_{X2}) \\ &\quad + 2a_1 a_3 (X_1 - \mu_{X1})(X_3 - \mu_{X3}) \\ &\quad + 2a_2 a_3 (X_2 - \mu_{X2})(X_3 - \mu_{X3})]\end{aligned}$$

or

$$\begin{aligned}\sigma_Y^2 &= a_1^2 \sigma_{X1}^2 + a_2^2 \sigma_{X2}^2 + a_3^2 \sigma_{X3}^2 \\ &\quad + 2a_1 a_2 \sigma_{X1X2} + 2a_1 a_3 \sigma_{X1X3} + 2a_2 a_3 \sigma_{X2X3}\end{aligned}$$

or more generally, the variance of Y will be given by

$$\sigma_Y^2 = \sum_{i=1}^{n} \left(a_i^2 \sigma_{Xi}^2 + 2 \sum_{j=i+1}^{n} a_i a_j \sigma_{XiXj} \right)$$

C.4 PROPAGATION OF UNCERTAINTY/ERROR THROUGH TAYLOR SERIES

In para. C.3 we found that if a random variable, Y, could be expressed as a linear combination of the random variables, X_i, the mean and variance of Y can be expressed in terms of the means, variances, and covariances of the variables X_i. Now suppose an experimental result, r, is defined as a function of certain measured variables, $X_1, X_2, \ldots, X_n$. If the function $r(X_1, X_2, \ldots, X_n)$ can be expressed as a linear combination of the measured variables $X_1, X_2, \ldots, X_n$, by means of a Taylor series approximation to $r(X_1, X_2, \ldots, X_n)$ in the neighborhood of $\mu_{X1}, \mu_{X2}, \ldots, \mu_{Xn}$, then using the results in para. C.3 we will be able to express the mean and variance of $r(X_1, X_2, \ldots, X_n)$ in terms of the means, variances, and covariances of the variables $X_1, X_2, \ldots, X_n$.

C.4.1 The First Order Approximation. Suppose $r = r(x_1, x_2, \ldots, x_n)$. If we expand r through a Taylor series in the neighborhood of $\mu_{X1}, \mu_{X2}, \ldots, \mu_{Xn}$ we get

$$\begin{aligned}r(x_1, x_2, \ldots, x_n) &= r(\mu_{X1}, \mu_{X2}, \ldots, \mu_{Xn}) + \theta_{X1} (x_1 \\ &\quad - \mu_{X1}) + \theta_{X2} (x_2 - \mu_{X2}) + \ldots + \theta_{Xn} (x_n - \mu_{Xn}) \\ &\quad + \text{higher order terms}\end{aligned}$$

where θ_{xi} are the sensitivity coefficients given by

$$\theta_{Xi} = \frac{\partial r}{\partial x_i}$$

Now, suppose that the arguments of the function $r(X_1, X_2, \ldots, X_n)$ are the random variables $X_1, X_2, \ldots, X_n$. Furthermore, assume that the higher order terms in the Taylor series expansion for r are negligible compared to the first order terms. Then in the neighborhood of $\mu_{X1}, \mu_{X2}, \ldots, \mu_{Xn}$ we have

$$\begin{aligned}r(X_1, X_2, \ldots, X_n) &\approx r(\mu_{X1}, \mu_{X2}, \ldots, \mu_{Xn}) + \theta_{X1} (X_1 \\ &\quad - \mu_{X1}) + \theta_{X2} (X_2 - \mu_{X2}) + \ldots + \theta_{Xn} (X_n - \mu_{Xn})\end{aligned}$$

Consequently

$$\mu_r \approx r(\mu_{X1}, \mu_{X2}, \ldots, \mu_{Xn})$$

and

$$\sigma_r^2 \approx \sum_{i=1}^{n} \left(\theta_{Xi}^2 \sigma_{Xi}^2 + 2 \sum_{j=i+1}^{n} \theta_{Xi} \theta_{Xj} \sigma_{XiXj} \right)$$

For the special case where the random variables $X_1, X_2, \ldots, X_n$ are all independent we get

$$\sigma_r^2 \approx \sum_{i=1}^{n} \theta_{Xi}^2 \sigma_{Xi}^2$$

Table C.1 presents some useful formulas for propa-

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TABLE C.1
TAYLOR SERIES VARIANCE PROPAGATION FORMULAS

Function	Variance (in Absolute Units) and Absolute Sensitivities	Variance (Dimensionless) and Relative Sensitivities
$r = f(x, y)$	$S_r^2 = \left(\frac{\partial r}{\partial x} S_x\right)^2 + \left(\frac{\partial r}{\partial y} S_y\right)^2$	$V_r^2 = \left(\frac{\partial r/r}{\partial x/x} V_x\right)^2 + \left(\frac{\partial r/r}{\partial y/y} V_y\right)^2$
	$\theta_x = \frac{\partial r}{\partial x}; \theta_y = \frac{\partial r}{\partial y}$	$\theta_x' = \frac{\partial r/r}{\partial x/x}; \theta_y' = \frac{\partial r/r}{\partial y/y}$
$r = Ax + By$	$S_r^2 = A^2 S_x^2 + B^2 S_y^2$	$V_r^2 = \frac{A^2 x^2 V_x^2 + B^2 y^2 V_y^2}{(Ax + By)^2}$
	$\theta_x = A; \theta_y = B$	
		$\theta_x' = \frac{Ax}{Ax + By}; \theta_y' = \frac{By}{Ax + By}$
$r = \frac{1}{y}$	$S_r^2 = \frac{S_y^2}{y^4}$	$V_r^2 = V_y^2$
	$\theta_y = -\frac{1}{y^2}$	$\theta_y' = -1$
$r = \frac{x}{x + y}$	$S_r^2 = \left[\frac{yS_x}{(x + y)^2}\right]^2 + \left[\frac{xS_y}{(x + y)^2}\right]^2$	$V_r^2 = \frac{y^2(V_x^2 + V_y^2)}{(x + y)^2}$
	$\theta_x = \frac{y}{(x + y)^2}; \theta_y = -\frac{x}{(x + y)^2}$	$\theta_x' = \frac{y}{x + y}; \theta_y' = -\frac{y}{x + y}$
$r = \frac{x}{1 + x}$	$S_r^2 = \frac{S_x^2}{(1 + x)^4}$	$V_r^2 = \frac{V_x^2}{(1 + x)^2}$
	$\theta_x = \frac{1}{(1 + x)^2}$	$\theta_x' = \frac{1}{1 + x}$
$r = xy$	$S_r^2 = (yS_x)^2 + (xS_y)^2$	$V_r^2 = V_x^2 + V_y^2$
	$\theta_x = y; \theta_y = x$	$\theta_x' = 1; \theta_y' = 1$
$r = x^2$	$S_r^2 = 4x^2 S_x^2$	$V_r^2 = 4V_x^2$
	$\theta_x = 2x$	$\theta_x' = 2$
$r = x^{1/2}$	$S_r^2 = \frac{S_x^2}{4x}$	$V_r^2 = \frac{V_x^2}{4}$
	$\theta_x = \frac{1}{2x^{1/2}}$	$\theta_x' = \frac{1}{2}$
$r = \ln x$	$S_r^2 = \frac{S_x^2}{x^2}$	$V_r^2 = \left(\frac{V_x}{\ln x}\right)^2$
	$\theta_x = \frac{1}{x}$	$\theta_x' = \frac{1}{\ln x}$

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TABLE C.1
TAYLOR SERIES VARIANCE PROPAGATION FORMULAS (CONT'D)

Function	Variance (in Absolute Units) and Absolute Sensitivities	Variance (Dimensionless) and Relative Sensitivities
$r = kx^a y^b$	$S_r^2 = (aky^b x^{a-1} S_x)^2 + (bkx^a y^{b-1} S_y)^2$	$V_r^2 = (aV_x)^2 + (bV_y)^2$
	$\theta_x = aky^b x^{a-1};\ \theta_y = bkx^a y^{b-1}$	$\theta_x' = a,\ \theta_y' = b$

GENERAL NOTE:

$$V_x = \frac{S_x}{\bar{x}} \quad V_y = \frac{S_y}{\bar{y}} \quad V_r = \frac{S_r}{\bar{r}}$$

gating variance through the first order approximation to the Taylor series for an experimental result r.

C.4.2 Assessing the Validity of the First Order Approximation. In para. C.4.1 we assumed that the Taylor series expansion for r could be reasonably approximated through the first order terms. In this paragraph we will briefly assess the conditions under which this approximation is meaningful.

Let's first consider the simplest case where $r(x)$. Then

$$r(x) = r(\mu_x) + \theta_x (x - \mu_x) + (1/2!)\ \gamma_{xx} (x - \mu_x)^2 + \text{higher order terms}$$

where

$$\gamma_{xx} = \frac{\partial^2 r}{\partial x^2}$$

In this case the ratio of the second order term in the series to the first order term in the series is given by

$$R = \frac{\gamma_{xx} (x - \mu_x)}{\theta_x}$$

So that for this case the assumption that $R \ll 1$ reduces to the condition that $\gamma_{xx} (x - \mu_x) \ll \theta_x$.

More generally, if the second order terms in the Taylor series expansion are retained, the Taylor series expansion for r in the neighborhood of $\mu_{x1}, \mu_{x2}, \ldots, \mu_{xn}$ becomes

$$r(X_1, X_2, \ldots, X_n) = r(\mu_{x1}, \mu_{x2}, \mu_{xn}) + \sum_{i=1}^{n} \theta_{xi} (x_i - \mu_{xi}) + (1/2!) \sum_{j=1}^{n} \sum_{k=1}^{n} \gamma_{xjxk} (x_j - \mu_{xj})(x_k - \mu_{xk}) + \text{higher order terms}$$

where

$$\gamma_{xjxk} = \frac{\partial^2 r}{\partial x_j \partial x_k}$$

The second order terms in this expansion may be compared directly to the first order terms in order to assess their significance.

C.4.3 The Limitation of the Present Approach. If the higher order terms in the Taylor series expansion are not small compared to the first order terms, then there is no way to express the variance in $r(X_1, X_2, \ldots, X_n)$ directly in terms of the variances and covariances for $X_1, X_2, \ldots, X_n$ for an arbitrary joint probability density function $p(x_1, x_2, \ldots, x_n)$.

C.5 PROPAGATION OF SYSTEMATIC AND RANDOM COMPONENTS OF UNCERTAINTY

The total variance associated with a measured variable, X_i, can be expressed as a combination of the variance associated with a fixed component and the variance associated with a random component of the total error in the measurement. In this paragraph we will relate these two sources of variance in the measured variables, X_i, to the variance in an experimental result $r(X_1, X_2, \ldots, X_n)$.

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In para. 4.2, the total error in a measurement was given by

$$\delta = \beta + \epsilon$$

so that

$$E[\delta] = E[\beta + \epsilon]$$
$$\mu_\delta = \mu_\beta$$

also

$$E[(\delta - \mu_\delta)^2] = E[\{(\beta - \mu_\beta) + \epsilon\}^2]$$
$$\sigma_\delta^2 = \sigma_\beta^2 + \sigma_\epsilon^2 + 2\sigma_{\beta\epsilon}$$

Assuming the systematic error to be independent of the random component of error this becomes

$$\sigma_\delta^2 = \sigma_\beta^2 + \sigma_\epsilon^2$$

Now, if we define the random variable X_i as follows,

$$X_i = (\delta_i - \mu_{\beta i}) + \mu_{Xi}$$

then

$$E[X_i] = \mu_{Xi}$$
$$E[(X_i - \mu_{Xi})^2] = \sigma_{\delta i}^2 = \sigma_{\beta i}^2 + \sigma_{\epsilon i}^2 = \sigma_{Xi}^2$$

Assuming fixed errors to be independent of random errors, it can also be shown that

$$E[(X_i - \mu_{Xi})(X_j - \mu_{Xj})] = \sigma_{XiXj} = \sigma_{\beta i\beta j} + \sigma_{\epsilon i\epsilon j}$$

These results can be combined with those in para. C.3.1 to yield

$$\sigma_r^2 \approx \sum_{i=1}^{n} \left(\theta_{Xi}^2 \sigma_{\epsilon i}^2 + 2 \sum_{j=i+1}^{n} \theta_{Xi}\theta_{Xj}\sigma_{\epsilon i\epsilon j} \right) + \sum_{i=1}^{n} \left(\theta_{Xi}^2 \sigma_{\beta i}^2 + 2 \sum_{j=i+1}^{n} \theta_{Xi}\theta_{Xj}\sigma_{\beta i\beta j} \right)$$

For no correlation among the random errors, with the use of sample statistics, and for a 95% confidence level, this equation leads to Eq. (B.3).

C.6 THE PROBABILITY DENSITY FUNCTION OF A RESULT

The preceding paragraphs make no assumptions about the joint probability density function of the measured variables X_i. Assuming that the first order approximation to the Taylor series expansion for r is adequate and that the measured variables X_i are jointly normally distributed, the experimental result r will also be normally distributed with mean μ_r and variance σ_r^2.

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APPENDIX D — UNCERTAINTY OF DERIVED RESULTS ON X-Y PLOTS

(This Appendix is not a part of ASME PTC 19.1-1998 and is included for information purposes only.)

D.1 INTRODUCTION

The final product of a test is frequently a plot of two results which have been calculated, or derived, from measured parameters. This plot, with derived results on both the ordinate (Y-axis) and abscissa (X-axis) will be influenced by errors in the measurements. The purpose of this Appendix is to introduce a simple methodology for propagation of measured parameter uncertainty to an X-Y plot of derived results. This methodology can be used to estimate the net uncertainty of a derived Y result for a specific X value given the uncertainty of the measurements.

Procedures for propagating uncertainty of measurements to individual, isolated derived results are well established. However, derived results often have little meaning in isolation. Gas turbine engine performance plots, for example, almost always contain derived results on both axes. A plot such as Corrected Fuel Flow (ordinate) versus Corrected Net Thrust (abscissa) may be the ultimate product of a test; the criterion to accept or reject an engine. And errors in the measurements will affect both derived results and, consequently, the relationship between the two. Unfortunately, it is not always possible to develop plots with an independent, errorfree variable on the abscissa. Therefore, a technique is required to propagate uncertainty information of the measurements to a plot of derived results. Uncertainty of the derived results in isolation is irrelevant.

Occasionally, both derived results on the plot are partially dependent on the same measurements. In the Corrected Fuel Flow versus Corrected Net Thrust example, inlet temperature and inlet pressure affect both results. Uncertainty in these common parameters can have a profound effect on the net uncertainty of the plot. This procedure is sufficiently rigorous to accommodate common parameters. The concept of a net uncertainty interval for a plot of derived results is displayed in Fig. D.1.

D.2 DEFINITIONS OF RANDOM AND SYSTEMATIC ERROR

Before an error source can be classified as contributing to the random or systematic uncertainty of an X-Y plot, the test result must be clearly defined. For this process, the test result is defined as the curve fit relationship for the Y result as a function of the X result. An equation defining Y as a function of X may take the form: $Y = a + bX + cX^2 + \ldots zX^n$.

With this definition of test result, random error is defined as the scatter about the curve fit line. Systematic error is defined as the difference, in the Y direction, between the curve fit line, generated from the data, and the true relationship between the X and Y results. The two error classifications are shown in Fig. D.2.

The Least Squares Regression technique is typically used to determine the equation for Y as a function of X. Many existing data analysis software packages are available to generate the optimum curve fits.

D.3 RANDOM UNCERTAINTY

Random error, commonly called precision error, is readily observed as data scatter about the curve fit line. In addition to the variability in the measured parameters, the random error of the curve fit may include additional effects such as the stability and hysteresis of the test article and the repeatability of the test process.

Random uncertainty, an estimate of the true random error, is comprised of two terms: the Residual Standard Deviation ($S_{Y/X}$) and the sample size for the number of data points N about the curve fit

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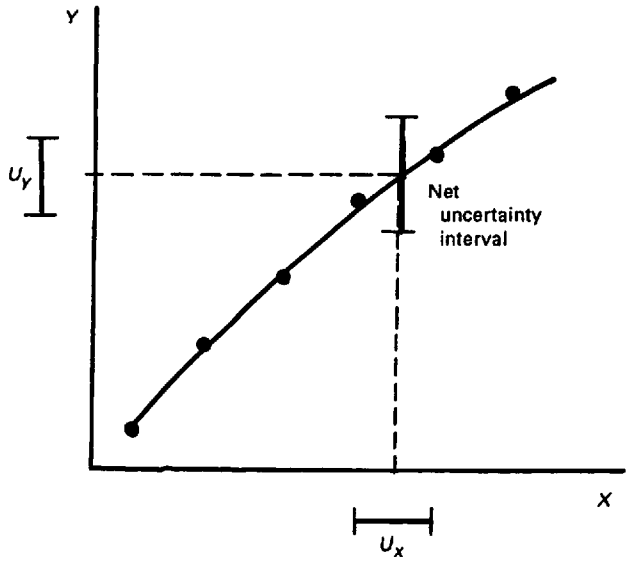

FIG. D.1 NET UNCERTAINTY INTERVAL

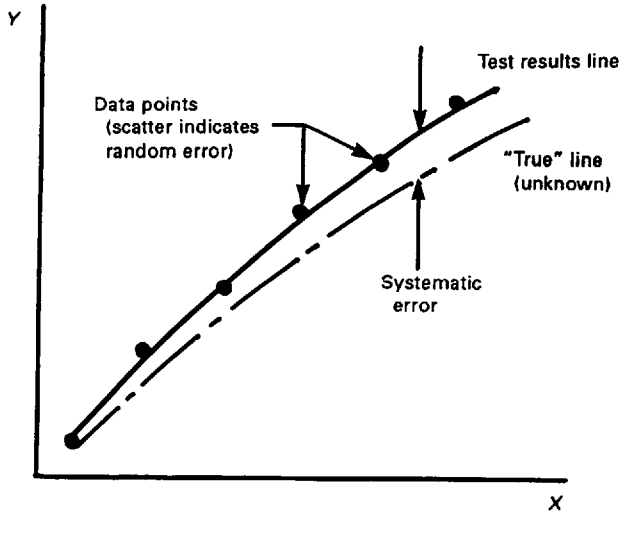

FIG. D.2 RANDOM AND SYSTEMATIC ERROR

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relationship. An approximate expression for random uncertainty is

$$\frac{S_{Y|X}}{\sqrt{N}}$$

D.3.1 Residual Standard Deviation ($S_{Y|X}$)

The Residual Standard Deviation is simply the standard deviation of the differences between the observed Y value (Y_{obs}) and the curve fit value (Y_{curv}) at each X data point. The Residual Standard Deviation is given by

$$S_{Y|X} = \sqrt{\frac{\sum_{i=1}^{N} (Y_{obs} - Y_{curv})^2}{N - C}}$$

where N is the number of data points used to generate the line and C is the number of coefficients in the curve fit equation.

D.3.2 Qualifying Assumptions

(a) The Residual Standard Deviation is a constant value for the entire range of the curve fit equation. However, the commonly used Least Squares regression technique produces a relationship that is more precise at the mean X value than at the extremes. Accordingly, the Residual Standard Deviation should be modified at the desired X value (x^*). For a linear Least Squares regression, the random uncertainty is given by

$$S_{Y|X} \sqrt{\frac{1}{N} + \frac{(x^* - \bar{x})^2}{\sum_{i=1}^{N} (x_i - \bar{x})^2}}$$

For higher order regression techniques, the terms under the square root become significantly more complex and can be found in advanced statistics handbooks. However, since this effect is usually quite small, the complexity has been dropped to simplify the equation to $1/N$. It is left to the reader to decide if the magnitude of these adjustments is sufficient to warrant the added complexity.

(b) In calculating the Residual Standard Deviation, the abscissa (X) is assumed to have no variability. Realistically, we recognize that on a plot of derived results, both results will contain random errors. To use the Residual Standard Deviation as an estimate of the random error, we must assume that the random error in the abscissa is "embedded" in the ordinate (Y). This assumption greatly simplifies the procedure for quantifying the random uncertainty of the curve fit relationship.

D.4 SYSTEMATIC UNCERTAINTY

Systematic error, or bias error, is defined as the difference, in the Y direction, between the observed curve fit relationship obtained from the test results and the true relationship. An upper and lower limit of the true systematic error can only be estimated since the true relationship is not precisely known. This estimate, based on engineering judgment, is termed the net systematic uncertainty.

The magnitude of the net systematic uncertainty, which may vary across the range of X values, is based on the magnitude of the systematic uncertainties of the measured parameters contained in the derived X and Y results. The contribution of each measured parameter systematic uncertainty to the Y result must be propagated individually and is a function of the measured parameter systematic uncertainty, the sensitivities of the derived results to the measured parameters, and partial derivative of the derived Y result to the derived X result (slope). The general expression for the contribution of each measured parameter systematic uncertainty to the plot is given by

$$B_{P_i} = B_i \left(\frac{\partial y}{\partial i} - \frac{\partial x}{\partial i} \frac{dy}{dx} \right)$$

where

$B_{P,i}$ = net systematic uncertainty of the plot for measured parameter i

B_i = systematic uncertainty of measured parameter i

$\frac{dy}{dx}$ = sensitivity of result Y to result X (slope)

$\frac{\partial y}{\partial i}$ = sensitivity of result Y to measured parameter i

$\frac{\partial x}{\partial i}$ = sensitivity of result X to measured parameter i

The propagation of measured parameter uncertainties to the net systematic uncertainty of the plot can be divided into three categories. First, some of the measured parameters affect only the Y result, shifting the curve up or down. Likewise, some of the measured parameters affect only the X result, shifting the curve left or right. And finally, some measured parameters may be contained in both the Y and X

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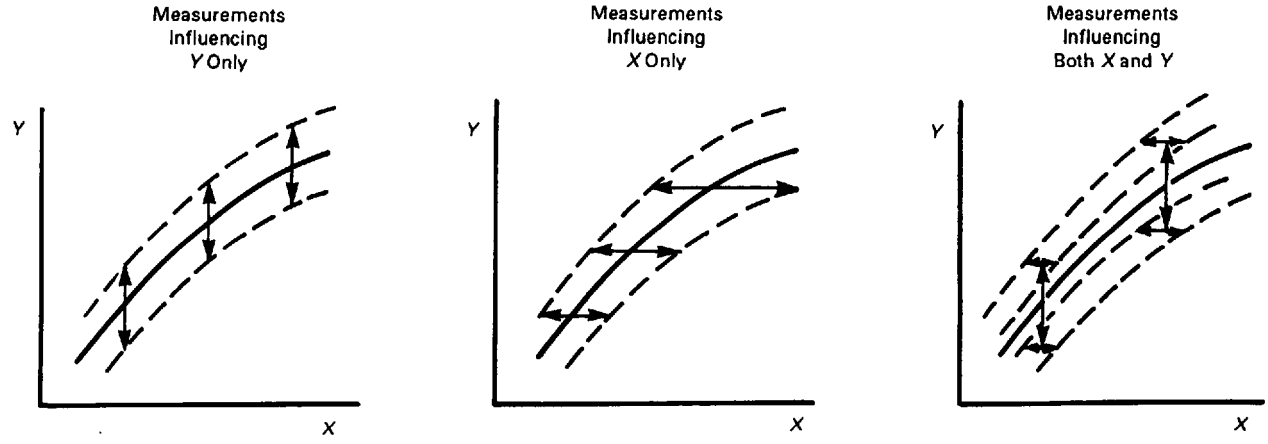

FIG. D.3 PROPAGATION PATHS OF MEASUREMENT ERRORS

derived results. Each of these propagation paths are presented graphically in Fig. D.3.

If all of the measured parameter systematic uncertainties are symmetric, the contributions are simply combined by root-sum-square to obtain the net systematic uncertainty for the plot.

$$B_P = \pm \sqrt{\sum_{i=1}^{J} B_{P_i}^2}$$

D.5 NET UNCERTAINTY INTERVAL

For simplicity of presentation, a single value is needed to express a reasonable limit for the maximum error between the curve fit line determined from measured data and the true line. Since the true line is unknown, it is convenient to define a limit referenced to the curve fit line. Uncertainty U is defined as an interval, in the Y direction for a specific X value, that has a preassigned probability of containing the true line. This uncertainty interval contains both the systematic and random uncertainty components.

The uncertainty interval of the plot for 95% coverage ($t = 2.0$) is calculated by

$$U_{95} = \pm 2.0 \sqrt{\left(\frac{B_P}{2.0}\right)^2 + \left(\frac{S_{Y|X}}{\sqrt{N}}\right)^2}$$

This equation is based on the following assumptions:

(a) a 95% coverage interval is appropriate;

(b) the degrees of freedom is sufficiently large that $t = 2.0$ can be used for the coverage multiplier;

(c) the measured parameter systematic uncertainty components are symmetric, normally distributed, and based on $2S$ elemental limits.

D.6 EXAMPLE: GAS TURBINE ENGINE PERFORMANCE

Problem description: Determine the uncertainty of engine corrected fuel flow at a specified corrected HP rotor speed.

Definition and value of terms:

WFRD	Corrected fuel flow	810 g/s
NHRD	Corrected HP rotor speed	8820 rpm
wf	Measured fuel flow	800 g/s
nh	Measured HP rotor speed	9000 rpm
lhv	Fuel lower heating value	42750 J/g
t2av	Average station 2 total temperature	300°K
p2av	Average station 2 total pressure	98 kPa
B_{wf}	Fuel flow systematic uncertainty	±5 g/s
B_{nh}	HP rotor speed systematic uncertainty	±20 rpm
B_{lhv}	Lower heating value systematic uncertainty	±170 J/g
B_{t2}	Station 2 temperature systematic uncertainty	±1°K
B_{p2}	Station 2 pressure systematic uncertainty	±0.2 kPa
$S_{Y\|X}$	Residual standard deviation for the curve	10.0 g/s
N	Number of data points	10
C	Number of curve fit coefficients	3

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Procedure:

(a) Define the calculation routines for the results. The equations for corrected fuel flow and corrected HP rotor speed are

$$WFRD = \frac{wf \times 101.325 \times lhv}{p2av \times 42750 \times \sqrt{\frac{t2av}{288.15}}}$$

$$NHRD = \frac{nh}{\sqrt{\frac{t2av}{288.15}}}$$

(b) Determine the curve fit relationship. The curve fit equation for corrected fuel flow as a function of corrected high rotor speed is

$$WFRD = 5028 - 1.4264 \times NHRD + (1.075 \times 10^{-4}) \times NHRD^2$$

(c) Calculate the sensitivities (partial derivatives) at the point of interest. The sensitivities of corrected fuel flow and corrected HP rotor speed to the measured parameters are

$$\frac{\partial WFRD}{\partial wf} = 1.0125 \qquad \frac{\partial WFRD}{\partial lhv} = 0.01895$$

$$\frac{\partial WFRD}{\partial t2av} = -1.31 \qquad \frac{\partial WFRD}{\partial p2av} = -8.265$$

$$\frac{\partial NHRD}{\partial nh} = 0.980 \qquad \frac{\partial NHRD}{\partial t2av} = -14.66$$

(d) Determine the slope of the curve fit relationship at the point of interest. At 8820 rpm, the slope is

$$\frac{dWFRD}{dNHRD} = 0.470$$

(e) Determine the contributions from each measured parameter systematic uncertainties to the plot.

$$B_{P_{wf}} = B_{wf} \times \left(\frac{\partial WFRD}{\partial wf} - \frac{\partial NHRD}{\partial wf} \times \frac{dWFRD}{dNHRD}\right)$$

$$B_{P_{wf}} = 5 \times [1.0125 - (0 \times .470)]$$

$$= \pm 5.06 \text{ g/s}$$

$$B_{P_{lhv}} = B_{lhv} \times \left(\frac{\partial WFRD}{\partial lhv} - \frac{\partial NHRD}{\partial lhv} \times \frac{dWFRD}{dNHRD}\right)$$

$$B_{P_{lhv}} = 170 \times [0.01895 - (0 \times 0.470)]$$

$$= \pm 3.22 \text{ g/s}$$

$$B_{P_{t2av}} = B_{t2av} \times \left(\frac{\partial WFRD}{\partial t2av} - \frac{\partial NHRD}{\partial t2av} \times \frac{dWFRD}{dNHRD}\right)$$

$$B_{P_{t2av}} = 1 \times [-1.31 - (-14.66 \times 0.470)]$$

$$= \pm 5.58 \text{ g/s}$$

$$B_{P_{p2av}} = B_{p2av} \times \left(\frac{\partial WFRD}{\partial p2av} - \frac{\partial NHRD}{\partial p2av} \times \frac{dWFRD}{dNHRD}\right)$$

$$B_{P_{p2av}} = 0.2 \times [-8.265 - (0 \times 0.470)]$$

$$= \pm 1.65 \text{ g/s}$$

$$B_{P_{nh}} = B_{nh} \times \left(\frac{\partial WFRD}{\partial nh} - \frac{\partial NHRD}{\partial nh} \times \frac{dWFRD}{dNHRD}\right)$$

$$B_{P_{nh}} = 20 \times [0 - (0.980 \times 0.470)]$$

$$= \pm 9.21 \text{ g/s}$$

(f) Combine measured parameter systematic uncertainty contributions to obtain net systematic uncertainty for the plot.

$$B_P = \pm \sqrt{\sum_{i=1}^{j} (B_{P_i})^2}$$

$$B_P = \pm \sqrt{5.06^2 + 3.22^2 + 5.58^2 + 1.65^2 + 9.21^2}$$

$$= 12.4 \text{ g/s}$$

(g) Combine systematic and random estimates into net uncertainty.

$$U_{95} = \pm 2.0 \sqrt{\left(\frac{B_P}{2.0}\right)^2 + \left(\frac{S_{Y|X}}{\sqrt{n}}\right)^2}$$

$$U_{95} = \pm 2.0 \sqrt{\left(\frac{12.4}{2.0}\right)^2 + \left(\frac{10.0}{\sqrt{10}}\right)^2}$$

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$$U_{95} = \pm 13.9 \text{ g/s}$$

At a corrected HP rotor speed of 8820 rpm, the corrected fuel flow is

$$810 \pm 13.9 \text{ g/s}$$

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